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# MECHANICAL ENGINEERING

# THERMAL AND FLUID SYSTEMS

# STUDY PROBLEMS

# PRINCIPLES:

# THERMODYNAMICS & ENERGY BALANCES

**How to use this book**

The exam specifications in effect since April 2017 state that approximately 6 problems from the "Thermodynamics" topic will be in the morning "Principles" portion of your exam. The problems in this book will prepare you for all the thermodynamics problems in the morning "Principles" portion. Furthermore, a solid understanding of this topic is required to have any success with the 24 questions in the afternoon "Energy/Power System Applications" portion. The problems in this book will prepare you for the thermodynamics problems of the afternoon "Energy/Power System Applications" portion of the exam.

**How it works**

This study problems book works on what we call the "principle of progressive overload". With this technique you start with very easy problems and smoothly progress towards more complex problems. A good example of progressive overload is the story of the famous wrestler Milo of Croton in ancient Greece. This extraordinarily strong man was allegedly capable of carrying a fully grown bull on his shoulders. He was reported to have achieved this tremendous strength by walking around town with a new born calf on his shoulders every single day. As the calf grew, so did the man's strength.

We recommend you work the problems in this book in the order they are presented. Within each section of the book, the first problems will feel "light", like carrying that baby calf – you might even be tempted to skip them. We strongly urge you to resist this temptation. As you progress, the problems become harder, but the work you've been putting in with all the previous problems will bear fruit. You will be pleasantly surprised at how relatively easy those "hard" problems will seem. You will soon be carrying intellectual bulls on your shoulders! **The problems that are considered "exam-level difficult" are denoted with an asterisk**.

This book is comprised of the following sections:

For the most part, these sections are not independent (specially the first seven) and build from the previous ones. We recommend you go through them in the order presented, and be sure to review them all. Each section begins with a brief discussion of the relevant concepts and equations. These discussions are laser-focused on the aspects that are relevant to the P.E. exam and do not go into derivations with academic rigor.

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# SECTION 01: Mass and Volume Flow Rates

The key equation for this section is the relationship between mass flow rate, $\dot{m}$ , volume flow rate, $\dot{V}$ , and average flow velocity, $V$ . This relationship takes on many forms, but they are all really the same:

$$\dot{m}=\rho\,A\,V \tag{1-1}$$

Since:

$$\dot{V}=A\,V \tag{1-2}$$

where $A$ is the pipe or duct cross-sectional area, we also have the alternate forms of equation (1-1): $\dot{m}=\rho\,\dot{V}$ , $\dot{m}=\dot{V}/v$ , and $\dot{m}=A\,V/v$ , where $\rho$ is density, and $v=1/\rho$ is the specific volume.

## PROBLEMS

**01-01.** Water ( $\rho=62.4\,\mathrm{lbm/ft}^3$ ) flows at a rate of 300 gpm in a 4-inch, schedule-40, seamless steel pipe (ID=4.026 inches). What is the average flow velocity, in feet per minute (fpm)?

**01-02.** Water ( $\rho=62.4\,\mathrm{lbm/ft}^3$ ) flows at a rate of 300 gpm in a 4-inch, schedule-40, seamless steel pipe (ID=4.026 inches). What is the mass flow rate, in pound-mass per hour (lbm/h)?

**01-03.** Steam with a specific volume of $v=5\,\mathrm{ft}^3/\mathrm{lbm}$ flows at a velocity of 500 ft/s within a 4-inch, schedule-40, seamless steel pipe (ID=4.026 inches). What is the mass flow rate in lbm/h?

**01-04.** What is the volumetric flow rate of steam – in cubic feet per minute – for the previous problem?

**01-05**. Air flows in a circular cross section duct with a volumetric flow rate of 3,000 CFM (cubic feet per minute) and a flow velocity of 2,200 feet per minute.  What is the duct diameter, in inches?

**01-06.**  Which has a higher flow velocity in a 3-in ID pipe: 300 gpm of water, or 300 lbm/h of steam with $v=7\,\mathrm{ft}^3/\mathrm{lbm}$ ?

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# SECTION 02: Mass Balances

For the purposes of the P.E. Exam, the most general form of the mass balance equation is:

$$\frac{dM}{dT} = \sum_i \dot{m}_{\text{in},i} - \sum_j \dot{m}_{\text{out},j} \qquad (2\text{-}1)$$

That is, the rate at which the mass accumulates inside a control volume, $dM/dT$, is equal to total rate at which mass enters the control volume, $\sum \dot{m}_{\text{in},i}$ minus the total rate at which mass leaves the control volume, $\sum \dot{m}_{\text{out},j}$. We use the summation signs to account for multiple inlet and/or outlet ports.

When the total rates of mass into and out of the control volume are equal, there is no mass accumulation. This is the steady-state condition:

$$\sum_i \dot{m}_{\text{in},i} = \sum_j \dot{m}_{\text{out},j} \quad \text{(Steady State)} \qquad (2\text{-}2)$$

If there is only one inlet and one outlet in the control volume, then:

$$\dot{m}_{\text{in}} = \dot{m}_{\text{out}} \qquad \text{(Steady state, one inlet, one outlet)} \qquad (2\text{-}3)$$

For incompressible fluids (liquids, or gases that do not experience significant density changes) the above equations can all be written in terms of volume and volumetric flow rates as well.

## PROBLEMS

**02-01.** Water ( $\rho = 62.4\,\text{lbm}/\text{ft}^3$ ) flows at a rate of 300 gpm in a 4-inch, schedule-40, seamless steel pipe (ID=4.026 inches). The pipe is reduced down to a 3-inch, schedule-40, seamless steel pipe (ID=3.068 inches). What is the percentage increase in average flow velocity after the reduction?

**02-02.** Air flows in a circular cross section duct containing a heating section. The volumetric flow rate upstream of the heating section is 3,000 CFM. The air density changes from $\rho_{\text{cold}} = 0.075\,\text{lbm}/\text{ft}^3$ to $\rho_{\text{hot}} = 0.06\,\text{lbm}/\text{ft}^3$ across the heating section. What is the volumetric flow rate of air downstream of the heating section?

**02-03.** A stream of 8 kg/s of liquid ammonia enters a throttle valve in which the pressure drops and some of the ammonia flashes into vapor. If the flow is at steady state, what is the mass flow rate of the liquid-vapor ammonia mixture leaving the throttle valve?

**02-04.** A stream of 300 gpm of cold water and another stream of 400 gpm of hot water enter a mixing chamber. If the chamber operates at steady state, and has only one outlet, what is the water flow rate (gpm) at the discharge?

**02-05\*.** A stream of $0.019\,\mathrm{m^3/s}$ of cold water and another stream of $0.025\,\mathrm{m^3/s}$ of hot water enter a mixing chamber. Both inlet pipes have an internal diameter of 77 mm. The chamber operates at steady state, and has only one outlet. It is required that the flow velocity at the discharge must not exceed the flow velocity of the cold water inlet. The smallest allowed internal diameter (mm) for the outlet pipe is most nearly:

(A) 0.117

(B) 77

(C) 117

(D) 234

**02-06\*.** A hydraulic jump is in place downstream from a spillway as indicated in the figure depicting a channel of constant width. The depth of the water upstream of the jump is 6 inches and the average stream velocity is 12 ft/s. Just downstream of the jump, the average stream velocity is 3 ft/s. The water depth (ft) just downstream of the jump is most nearly:

(A) 1.5

(B) 2

(C) 24

(D) 48
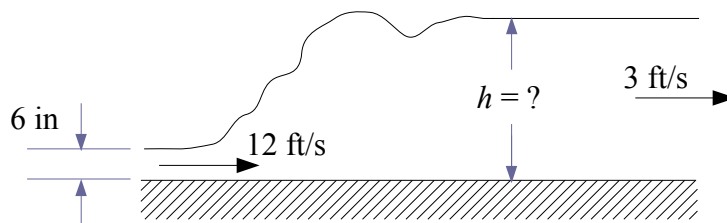
**02-07\*.** An airplane jet engine receives air at a rate of 70 lbm/s. Simultaneously, fuel enters the engine at a steady rate of 0.55 lbm/s. The average velocity of the exhaust gases is 1,550 ft/s relative to the engine. The engine exhaust effective cross-sectional area is 3.5 ft$^2$. The density (lbm/ft$^3$) of the exhaust gases is most nearly:

(A) 0.005

(B) 0.08

(C) 0.010

(D) 0.013

**02-08\*** The evaporative cooling tower shown in the figure is used to cool water (which enters at a rate of 250,000 lbm/h) from 100°F to 80°F. Dry air (no water vapor) flows into the tower at a rate of 150,000 lbm/h. The rate of wet air flow out of the tower is 155,900 lbm/h. Because some of the hot water evaporates, some make-up water has to be added so the flow rate of cooled water leaving the tower equals the rate of hot water entering. The required make-up water flow rate (lbm/h) is most nearly:

(A) 5,900

(B) 155,900

(C) 244,100

(D) Can't be determined
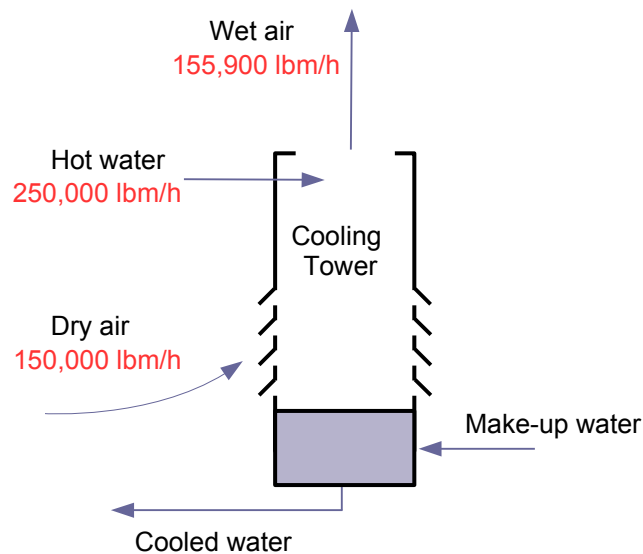
**02-09\*** An above-ground water storage tank has a circular cross section with a diameter of 33 ft and a height of 16 ft. Initially, the tank is 10% liquid full. When a loading operation starts, the water is pumped into the tank at a rate of 1,100 gpm. The high-level alarm is triggered when the liquid level reaches 14 ft. The elapsed time (minutes) between beginning loading and triggering the high-level alarm is most nearly:

(A) 20

(B) 40

(C) 60

(D) 80

# SECTION 03: Introduction to Energy Balances

Nearly all the thermodynamics problems in the P.E. Exam can be solved by performing one or more energy balances (The energy balance is also known as the first law of thermodynamics). An energy balance consists simply of accounting for all forms of energy that enter or leave a certain control volume. For purposes of the P.E. Exam, the forms of energy that typically enter or leave a control volume are <u>work</u>, <u>heat</u> and <u>energy carried by fluids</u>.

Work enters or leaves a control volume through a rotating shaft (mechanical work) and/or electrical conductors (electrical work). The *rate* at which work enters or leaves a control volume is *power*.

Heat enters or leaves a control volume whenever the surface of the control volume is exposed to surroundings that are at a different temperature than the control volume surface. The *rate* at which heat enters or leaves a control volume is simply the *heat transfer rate*.

Finally, energy enters or leaves a control volume with any stream of fluid that enters or leaves the control volume. On a per-unit mass basis, the amount of energy carried into or out of a control volume by a stream of fluid is:

$$e_{\text{fluid}} = u + P v + \frac{V^2}{2} + g z$$

The last two terms are the familiar kinetic and potential energy terms. The first term, $u$, is the *internal energy*,[1] while the second term, $P v$, is known as the *flow work*[2]. The group $u + P v$ shows up so often when analyzing control volumes in thermal fluids engineering that they are lumped together and given a special name, *enthalpy*, $h = u + P v = u + P / \rho$.

For the purposes of the P.E. Exam, energy balances are performed on devices operating at steady state, and the most general form of the mass balance equation is:

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}} \tag{3-1}$$

That is, the rate at which energy (in *all its forms*) enters a control volume must be the same as the rate at which it leaves the control volume. For a control volume with an arbitrary number of mass flow inlets and outlets, the generic energy balance is:

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1   Internal energy is the total sum of several microscopic forms of energy related to the molecular structure of a system and the degree of the molecular activity.

2   It can be shown that flow work is the energy required to push mass into or out of a control volume. This work is necessary for maintaining a continuous flow through a control volume.

$$\dot{W}_{in} + \dot{Q}_{in} + \sum_i \dot{m}_{in,i}\left(h + \frac{V^2}{2} + gz\right)_i = \dot{W}_{out} + \dot{Q}_{out} + \sum_j \dot{m}_{out,j}\left(h + \frac{V^2}{2} + gz\right)_j \qquad (3\text{-}2)$$

In the left hand side of (3-2) we are adding all the rates of energy coming in in the form of work (could be electrical and/or mechanical), heat, and that brought in with fluid flow in each of the flow inlets respectively. All this must be equal to the sum of all the rates at which energy is leaving the control volume.

In the typical P.E. Exam problem, most terms in equation (3-2) are either negligible or cancel each other out and thus need not be considered. Which terms can be crossed out depends on the particular application.

Consider for example an industrial continuous flow heater, with one inlet and one outlet.

We define the control volume with the red dashed line. A cold stream with a mass flow rate $\dot{m}$ is continuously flowing into the heater, and a hot stream of the same mass flow rate is continuously flowing out of it. The heater (the control volume) is losing heat to the surrounding air at a rate of $\dot{Q}_{out}$, and the electric heating element is supplying electrical work (heating)
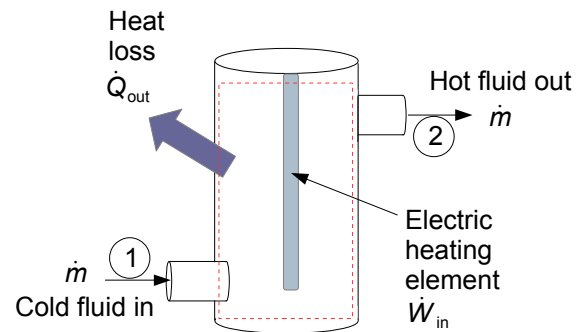


to the water at a rate of $\dot{W}_{in}$. For this example, the energy balance (3-2) reduces to:

$$\dot{W}_{in} + \dot{m}\left(h + \frac{V^2}{2} + gz\right)_1 = \dot{Q}_{out} + \dot{m}\left(h + \frac{V^2}{2} + gz\right)_2$$

which can be re-arranged as:

$$\dot{W}_{in} - \dot{Q}_{out} = \dot{m}\left[h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right]$$

On the basis of this equation, we can say that the fluid stream experiences an increase in its total energy as it flows through the heater that is equal to the electric energy supplied to the water minus the heat losses. If we divide through by $\dot{m}$, the above equation becomes:

$$\dot{w}_{in} - \dot{q}_{out} = \left[h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right]$$

where $\dot{w} = \dot{W}/\dot{m}$ and $\dot{q} = \dot{Q}/\dot{m}$ are the work and heat transfer per unit mass flow rate.

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This discussion of the fluid heater shows the typical terms that you will need to obtain in order to perform energy balances.

Consider the change in kinetic energy, $(V_2^2 - V_1^2)/2$. The SI unit of kinetic energy is m²/s², which is equivalent to a J/kg. The enthalpy is usually given in kJ/kg. To add these two quantities, the kinetic energy should be expressed in kJ/kg, so we divide it by 1000. A velocity change from rest to 45 m/s (100 mph!) corresponds to a kinetic energy increase of just 1 kJ/kg, which is a very small value compared with the enthalpy changes encountered in practice. In customary US units, a velocity change from rest to 225 ft/s (150 mph!) corresponds to a kinetic energy increase of only 1 Btu/lbm, which is a very small value compared with the enthalpy changes encountered in practice. Thus, the change in kinetic energy term at low velocities can be neglected. When a fluid stream enters and leaves a steady-flow device at about the same velocity, $V_1 \approx V_2$, the change in the kinetic energy is close to zero regardless of the magnitude of the velocity itself. With the exception of problems involving nozzles and diffusers, you can typically neglect the changes in kinetic energy in the thermodynamics section of the P.E. Exam.

A similar argument can be given for the potential energy change term, $g(z_2 - z_1)$. A potential energy change of 1 kJ/kg corresponds to an elevation difference of 102 m (335 ft), and a potential energy change of 1 Btu/lbm corresponds to an elevation difference of 780 ft. The elevation difference between the inlet and exit of most industrial devices such as turbines and compressors is well below this value, and the potential energy term is always neglected for these devices. The only time the potential energy term is significant is when a process involves pumping a fluid to high elevations and we are interested in the required pumping power (this kind of problem is the purview of the Hydraulic and Fluid Applications portion of the test).

Calculating the change in enthalpy, $(h_2 - h_1)$, is usually at the heart of the thermo problems in the test. How you calculate this term depends on the fluid you are dealing with. For liquids and ideal gases, $(h_2 - h_1)$ can be obtained with a simple analytical expression that depends on temperature. For greater accuracy an alternate approach involving tables can be used for gases. For liquid-vapor mixtures and superheated vapors, we rely on tables or graphs to look up individual values of enthalpy, in order to calculate $(h_2 - h_1)$. The next three sections are dedicated to studying how to calculate enthalpy changes

for different kinds of fluids. The review problems that follow are designed to get your toe wet with the energy balance without having to worry about calculating enthalpy changes. That is left for subsequent sections.

## Turbines & Compressors

In a steam, gas, or hydroelectric turbine, the fluid does work against the blades, which are attached to a shaft. As a result, the shaft rotates, and the turbine produces work.

Compressors, as well as pumps and fans, are devices used to increase the pressure of a fluid, so these devices require work input. Even though these three devices function similarly, they do differ in the tasks they perform. A fan increases the pressure of a gas slightly and is mainly used to mobilize a gas. A compressor is capable of compressing the gas to very high pressures. Pumps work very much like compressors except that they handle liquids instead of gases.

Note that turbines produce power <u>output</u> whereas compressors, pumps, and fans require power <u>input.</u> Gas and steam turbines are typically well insulated so heat transfer from turbines is usually negligible. Heat transfer for compressors is also typically negligible, unless there is intentional cooling. Potential energy changes are negligible for all of these devices. The velocities involved in these devices, with the exception of turbines and fans, are usually too low to cause any significant change in the kinetic energy. The flow velocities encountered in most turbines are very high, and the fluid experiences a significant change in its kinetic energy. However, this change is usually very small relative to the change in enthalpy, and thus it is often disregarded.

## Throttling valves

Throttling valves (e.g. adjustable valves, porous plugs) are usually small devices, and the flow through them is assumed to be adiabatic (no heat loss to ambient) since there is neither sufficient time nor large enough area for any effective heat transfer to take place. Also, there is no work done, nor any significant change in potential energy. Even though the exit velocity is often considerably higher than the inlet velocity, the increase in kinetic energy is typically negligible. Then the conservation of energy equation for this one-inlet, one-outlet steady-flow device is simply $h_2 = h_1$.

That is, the enthalpy values at the inlet and exit of a throttling valve are the same. A throttling valve is sometimes called an *isenthalpic* device. Note, however, that for throttling devices with large exposed surface areas such as capillary tubes, heat transfer may be significant.

**Nozzles & Diffusers**

Nozzles and diffusers are commonly used in jet engines, spacecraft, and other applications requiring fluid jets. A nozzle increases the velocity of a fluid at the expense of pressure. A diffuser is a device that increases the pressure of a fluid by slowing it down. So, nozzles and diffusers perform opposite tasks. The cross-sectional area of a nozzle decreases in the flow direction for subsonic flows and increases for supersonic flows. The reverse is true for diffusers.
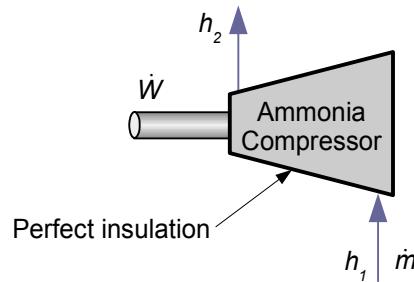
**PROBLEMS**

**03-01.** Write the energy balance for the turbine shown. *Hint*: List on one side of the equal sign all the ways that energy is entering the control volume, and on the other side list all the forms in which energy is leaving.
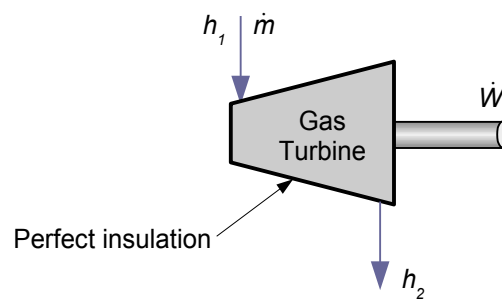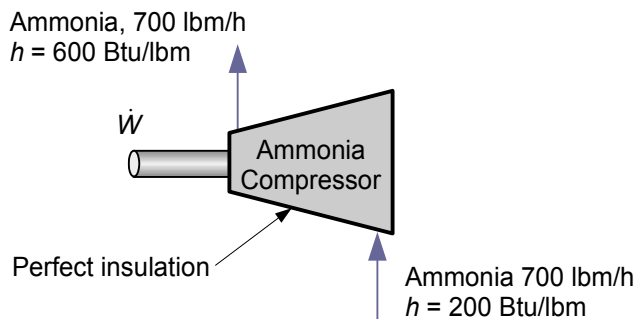


**03-02.** Write the energy balance for the turbine shown, and calculate the power output, in MW.
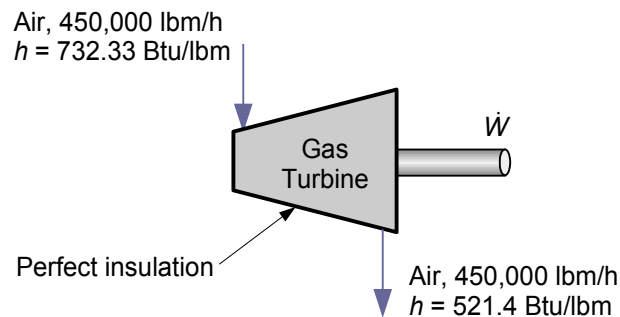
**03-03.** Write the energy balance for the compressor shown. *Hint*: List on one side of the equal sign all the ways that energy is entering the control volume, and on the other side list all the forms in which energy is leaving.



**03-04.** Write the energy balance for the compressor shown, and calculate the power consumption, in hp.



**03-05.** Write the energy balance for the compressor shown, and calculate the power consumption, in hp. *Hint*: List on one side of the equal sign all the ways that energy is entering the control volume, and on the other side list all the forms in which energy is leaving.
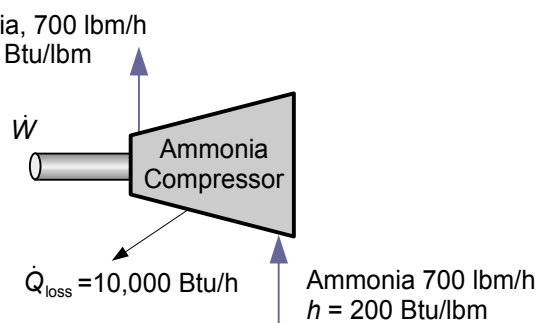
**03-06.** The figure shows a heat exchanger used as a condenser for superheated ammonia gas. The coolant is a stream of water. Over the course of several years, the insulation on the heat exchanger has been degraded so there is a non negligible amount of heat lost to the ambient. Write an energy balance on the heat exchanger to develop an expression for the heat transfer loss to ambient.
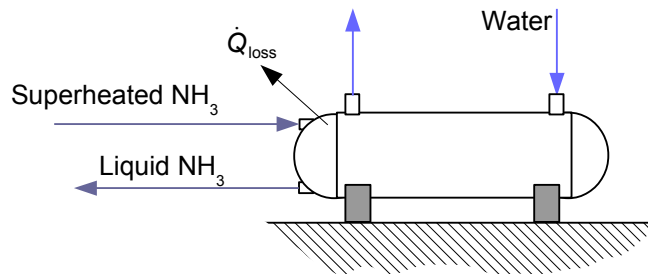


**03-07.** A desuperheater is a mixing chamber in which superheated steam is mixed with cold liquid water in the right proportion, to produce saturated vapor. Assume the chamber is perfectly insulated and write the energy balance and the mass balance, to obtain the required mass flow of liquid water into the desuperheater.
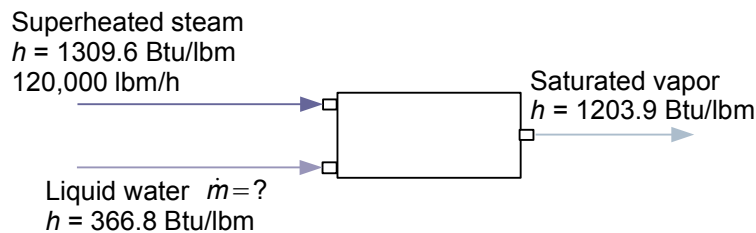


**03-08\*.** A turbine admits superheated steam at 750°F, $h$=1,400 Btu/lbm. Roughly 20% of the mass flow rate through the turbine is extracted at an intermediate step within the turbine, where pressure is atmospheric and $h$=1,200 Btu/lbm. The rest of the steam continues through the turbine until it is discharged at a pressure of 3 psia with $h$=1,100 Btu/lbm. If this is a 5 MW turbine, the mass flow rate (lbm per hour) of steam entering the turbine is most nearly:

(A) 20,000

(B) 40,000

(C) 60,000

(D) 80,000

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## SECTION 04: Control Volume Analysis – Incompressible Liquids

The previous sections presented energy balance and mass balance problems in which the enthalpies and/or densities (or specific volume) were given. In most problems of the P.E. exam this is information that will not be provided. This section and the next two are dedicated to reviewing how to obtain properties of fluids in order to solve energy balance problems. Calculating changes in enthalpy is at the heart of most thermo problems in the P.E. Exam.

For liquids, the key assumption is that they are incompressible. A substance whose specific volume (or density) is constant is called an **incompressible substance**. The specific volume of liquids (and solids) essentially remains constant during a process. Therefore, liquids (and solids) can be approximated as incompressible substances without loss of accuracy.

It can be shown that for incompressible substances, a change in enthalpy is produced whenever there are changes in temperature and/or pressure. The change in enthalpy for liquids is:

$$h_2 - h_1 = c\left(T_2 - T_1\right) + v\left(p_2 - p_1\right) \quad \text{(liquids)} \tag{4-1}$$

where $v$ is the specific volume, and $c$ is the specific heat. Strictly speaking, $c$ is a weak function of temperature. However, for the purposes of the P.E. Exam, assuming $c$ is a constant is a safe assumption.

For liquids, two common cases are typically encountered:

a) Constant pressure process (as in heat exchangers), then:

$$h_2 - h_1 = c\left(T_2 - T_1\right) \quad \text{(liquids in a constant } p \text{ process)} \tag{4-2}$$

b) Constant temperature process (as in pumps), then:

$$h_2 - h_1 = v\left(p_2 - p_1\right) \quad \text{(liquids in a constant } T \text{ process)} \tag{4-3}$$

Values of specific heats are widely available in the literature.

**PROBLEMS**

**04-01.** The continuous flow, well-insulated heater shown in the figure is used to heat 25,000 lbm/h of water, $c = 1\,\text{Btu}/(\text{lbm F})$, from 70°F to 140°F. Determine the power added by the heating element, in kW. *Hint*: start with an energy balance on the heater.

Hot fluid out
$\dot{m}$

Electric heating element

Cold fluid in
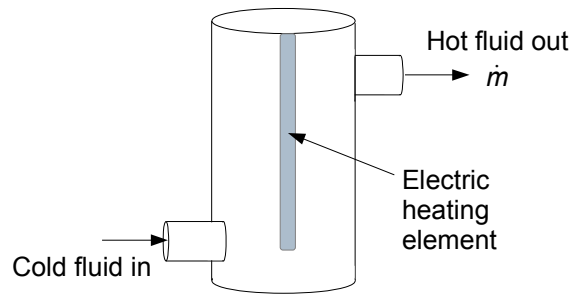
**04-02.** An alcohol solution of density 50 lbm/ft³ flows at a rate of 10 gpm through a centrifugal pump. A pressure gage at the suction indicates a vacuum of 0.25 psi and a gage at the discharge indicates a pressure of 10 psi.  At what rate is the pump adding energy to the fluid, in horsepower?

**04-03.** The heat exchanger shown uses water as the cooling fluid to cool and condense a stream of 1,000 lbm/h of superheated ammonia vapor. If the temperature rise for the water shall not exceed 20°F, the required flow rate (gpm) of water is most nearly:

(A) 57

(B) 225

(C) 4560

(D) 28,750

Superheated NH₃
$h$ = 725 Btu/lbm
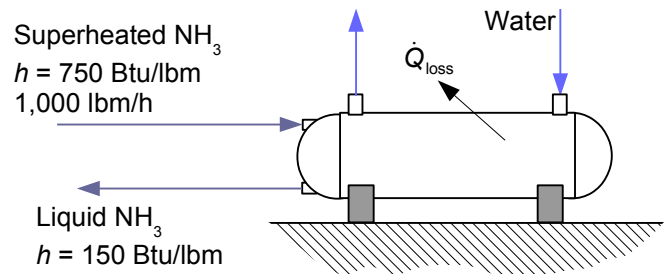1,000 lbm/h

Water

Liquid NH₃
$h$ = 150 Btu/lbm
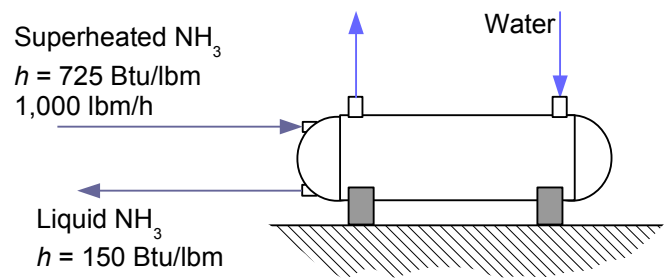
**04-04\*.** A poorly insulated continuous flow water heater is used for heating 50 gpm of water from 70ºF to 140ºF. The power consumed by the electric heating element is known to be 600 kW. The heat lost to the ambient (Btu/h) is most nearly:

(A) 25

(B) 87

(C) 330

(D) 513

**04-05\*.** The heat exchanger shown uses water as the cooling fluid to cool and condense a stream of 1,000 lbm/h of superheated ammonia vapor. The water flow rate is 60 gpm and the water temperature rise is 18ºF. The heat loss (Btu/h) to ambient from the vessel is most nearly:

(A) 8,750

(B) 12,000

(C) 60,000

(D) 575,000



Superheated NH$_3$
$h$ = 750 Btu/lbm
1,000 lbm/h

$\dot{Q}_{loss}$

Water
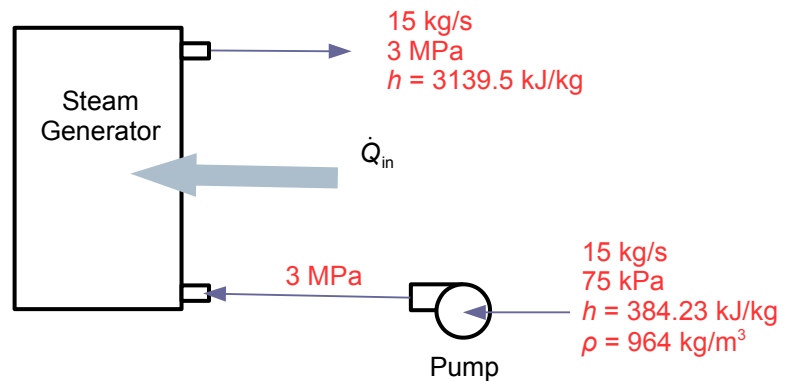
Liquid NH$_3$
$h$ = 150 Btu/lbm

**04-06\*.** A stream of 50,000 lbm/h of water at 190ºF is mixed with a stream of water at 70ºF in an adiabatic mixing chamber. The desired temperature of the discharge from the chamber is 100ºF. The required flow rate (gpm) of 70ºF water into the chamber is most nearly:

(A) 300

(B) 900

(C) 10,000

(D) 150,000

**04-07*** In a steam power plant, a pump raises the pressure of 15 kg/s of water from 75 kPa to 3 MPa. After the pump, the water enters a steam generator to be discharged as superheated steam. More data is provided in the figure. Under these conditions, the heat added by the steam generator, $\dot{Q}_{in}$ (MW) is most nearly:

(A) 4

(B) 41

(C) 412

(D) 41,200