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MECHANICAL ENGINEERING
THERMAL AND FLUID SYSTEMS
STUDY PROBLEMS

How to use this book

This study problems book works on what we call the "principle of progressive overload". With this technique you start with very easy problems and smoothly progress towards more complex problems. A good example of progressive overload is the story of the famous wrestler Milo of Croton in ancient Greece. This extraordinarily strong man was allegedly capable of carrying a fully grown bull on his shoulders. He was reported to have achieved this tremendous strength by walking around town with a new born calf on his shoulders every single day. As the calf grew, so did the man's strength.

We recommend you work the problems in this book in the order they are presented. Within each section of the book, the first problems will feel "light", like carrying that baby calf – you might even be tempted to skip them. We strongly urge you to resist this temptation. As you progress, the problems become harder, but the work you've been putting in with all the previous problems will bear fruit. You will be pleasantly surprised at how relatively easy those "hard" problems will seem. You will soon be carrying intellectual bulls on your shoulders! **The problems that are considered "exam-level difficult" are denoted with an asterisk**.

The book is divided into 5 parts with multiple sections within each part. Generally, the sections are <u>not</u> independent and build from the previous ones. We recommend you go through them in the order presented, and be sure to review them all. Each section begins with a brief discussion of the relevant concepts and equations. These discussions are laser-focused on the aspects that are relevant to the P.E. exam and do not go into derivations with academic rigor.

Starting in 2020, the only reference allowed in the exam is the NCEES PE Mechanical Handbook (PEMH) which you will have in the computer screen. We will use **bold text in blue** through this book whenever we refer to a table, equation, or graph from the PEMH.

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PART I: THERMODYNAMICS

01: Mass and Volume Flow Rates

The key equation for this section is the relationship between mass flow rate, $\dot{m}$, volume flow rate, $\dot{V}$, and average flow velocity, V. This relationship is known as **the continuity equation** and it takes on many forms, but they are all really the same:

$$\dot{m} = \rho\, A\, V \tag{1-1}$$

Since: $\dot{V} = A\, V$ where A is the pipe or duct cross-sectional area, we also have the alternate forms of Equation 1-1: $\dot{m} = \rho\, \dot{V}$, $\dot{m} = \dot{V}/v$, and $\dot{m} = A\, V/v$, where ρ is density, and $v = 1/\rho$ is the specific volume. In thermodynamics, we typically use $\dot{V}$ for volumetric flow rate whereas in hydraulics the symbol Q is used. In thermo, we reserve Q to denote heat transfer. **Equation 1-1 can be found in the PEMH by searching for "continuity equation"**

PROBLEMS

01-01. Water ($\rho = 62.4\,\text{lbm}/\text{ft}^3$) flows at a rate of 300 gpm in a 4-inch, schedule-40, seamless steel pipe (ID=4.026 inches). What is the average flow velocity, in feet per second?

01-02. Water ($\rho = 62.4\,\text{lbm}/\text{ft}^3$) flows at a rate of 300 gpm in a 4-inch, schedule-40, seamless steel pipe (ID=4.026 inches). What is the mass flow rate, in pound-mass per hour (lbm/h)?

01-03. Steam with a specific volume of $v = 5\,\text{ft}^3/\text{lbm}$ flows at a velocity of 500 ft/s within a 4-inch, schedule-40, seamless steel pipe (ID=4.026 inches). What is the mass flow rate in lbm/h?

01-04. What is the volumetric flow rate of steam – in cubic feet per minute – for the previous problem?

01-05. Air flows in a circular cross section duct with a volumetric flow rate of 3,000 CFM (cubic feet per minute) and a flow velocity of 2,200 feet per minute. What is the duct diameter, in inches?

01-06. Which has a higher flow velocity in a 3-in ID pipe: 300 gpm of water, or 300 lbm/h of steam with $v = 7\,\text{ft}^3/\text{lbm}$?

Solutions:

01-01.

$$
\begin{aligned}
V &= \frac{\dot{V}}{A} \\
&= \frac{300\,\dfrac{\text{gal}}{\text{min}}}{\dfrac{\pi}{4}\times(4.026\text{ in})^2} \\
&= \frac{300\,\dfrac{\text{gal}}{\text{min}}\times\left|\dfrac{1\text{ ft}^3}{7.48052\text{ gal}}\right|\times\left|\dfrac{1\text{ min}}{60\text{ s}}\right|}{\dfrac{\pi}{4}\times(4.026\text{ in})^2\times\left|\dfrac{1\text{ ft}^2}{144\text{ in}^2}\right|} = 7.56\,\frac{\text{ft}}{\text{s}}
\end{aligned}
$$

01-02.

$$
\begin{aligned}
\dot{m} &= \rho\,\dot{V} \\
&= 62.4\,\frac{\text{lbm}}{\text{ft}^3}\times 300\,\frac{\text{gal}}{\text{min}} \\
&= 62.4\,\frac{\text{lbm}}{\text{ft}^3}\times 300\,\frac{\text{gal}}{\text{min}}\times\left|\frac{1\text{ ft}^3}{7.48052\text{ gal}}\right|\times\left|\frac{60\text{ min}}{1\text{ h}}\right| = 150{,}150\,\frac{\text{lbm}}{\text{h}}
\end{aligned}
$$

01-03.

$$
\begin{aligned}
\dot{m} &= A\,V / v \\
&= \frac{\dfrac{\pi}{4}(4.026\text{ in})^2\times\left|\dfrac{1\text{ ft}^2}{144\text{ in}^2}\right|\times 500\,\dfrac{\text{ft}}{\text{s}}\times\left|\dfrac{3600\text{ s}}{1\text{ h}}\right|}{5\,\dfrac{\text{ft}^3}{\text{lbm}}} = 31{,}826\,\frac{\text{lbm}}{\text{h}}
\end{aligned}
$$

01-04.

$$
\begin{aligned}
\dot{V} &= v\,\dot{m} \\
&= 5\,\frac{\text{ft}^3}{\text{lbm}}\times 31{,}826\,\frac{\text{lbm}}{\text{h}} \\
&= 5\,\frac{\text{ft}^3}{\text{lbm}}\times 31{,}826\,\frac{\text{lbm}}{\text{h}}\times\left|\frac{1\text{ h}}{60\text{ min}}\right| = 2{,}652\,\frac{\text{ft}^3}{\text{min}}
\end{aligned}
$$

01-05.

$$A=\frac{\dot{V}}{V}$$

$$=\frac{3{,}000\,\frac{\text{ft}^3}{\text{min}}}{2{,}200\,\frac{\text{ft}}{\text{min}}}=1.364\ \text{ft}^2$$

$$D=\sqrt{\frac{4\,A}{\pi}}=\sqrt{\frac{4\times 1.364\ \text{ft}^2}{\pi}}\times\left|\frac{12\ \text{in}}{1\ \text{ft}}\right|=15.8\ \text{in}$$

01-06. Calculate both velocities (in the same units) in order to compare them:

$$V_{\text{water}}=\frac{\dot{V}_{\text{water}}}{A}=\frac{300\,\frac{\text{gal}}{\text{min}}}{\frac{\pi}{4}\times(3\ \text{in})^2}$$

$$=\frac{300\,\frac{\text{gal}}{\text{min}}\times\left|\frac{1\ \text{ft}^3}{7.48052\ \text{gal}}\right|}{\frac{\pi}{4}\times(3\ \text{in})^2\times\left|\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right|}$$

$$V_{\text{water}}=817\,\frac{\text{ft}}{\text{min}}$$

$$V_{\text{steam}}=\frac{\dot{m}_{\text{steam}}\,v}{A}=\frac{300\,\frac{\text{lbm}}{\text{h}}\times 7\,\frac{\text{ft}^3}{\text{lbm}}}{\frac{\pi}{4}\times(3\ \text{in})^2}$$

$$=\frac{300\,\frac{\text{lbm}}{\text{h}}\times 7\,\frac{\text{ft}^3}{\text{lbm}}\times\left|\frac{1\ \text{h}}{60\ \text{min}}\right|}{\frac{\pi}{4}\times(3\ \text{in})^2\times\left|\frac{1\ \text{ft}^2}{144\ \text{in}^2}\right|}$$

$$V_{\text{steam}}=713\,\frac{\text{ft}}{\text{min}}$$

Therefore, under the conditions described in the problem statement, the water is flowing with the highest average flow velocity.

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02: Mass Balances

For the purposes of the P.E. Exam, the most general form of the mass balance equation is:

$$\frac{dM}{dT}=\sum_{i}\dot{m}_{in,i}-\sum_{j}\dot{m}_{out,j} \qquad (2\text{-}1)$$

That is, the rate at which the mass accumulates inside a control volume, dM/dT , is equal to total rate at which mass enters the control volume, $\Sigma\dot{m}_{in,i}$ minus the total rate at which mass leaves the control volume, $\Sigma\dot{m}_{out,j}$. We use the summation signs to account for multiple inlet and/or outlet ports.

When the total rates of mass into and out of the control volume are equal, there is no mass accumulation. This is the steady-state condition:

$$\sum_{i}\dot{m}_{in,i}=\sum_{j}\dot{m}_{out,j} \quad \text{(Steady State)} \qquad (2\text{-}2)$$

If there is only one inlet and one outlet in the control volume, then:

$$\dot{m}_{in}=\dot{m}_{out} \quad \text{(Steady state, one inlet, one outlet)} \qquad (2\text{-}3)$$

For incompressible fluids (liquids, or gases that do not experience significant density changes) the above equations can all be written in terms of volume and volumetric flow rates as well. **Equation 2-2 can be found in the PEMH by searching for "steady-flow"**

PROBLEMS

02-01. Water ($\rho=62.4\,\text{lbm/ft}^3$) flows at a rate of 300 gpm in a 4-inch, schedule-40, seamless steel pipe (ID=4.026 inches). The pipe is reduced down to a 3-inch, schedule-40, seamless steel pipe (ID=3.068 inches). What is the percentage increase in average flow velocity after the reduction?

02-02. Air flows in a circular cross section duct containing a heating section. The volumetric flow rate upstream of the heating section is 3,000 CFM. The air density changes from $\rho_{cold}=0.075\,\text{lbm/ft}^3$ to $\rho_{hot}=0.06\,\text{lbm/ft}^3$ across the heating section. What is the volumetric flow rate of air downstream of the heating section?

02-03. A stream of 8 kg/s of liquid ammonia enters a throttle valve in which the pressure drops and some of the ammonia flashes into vapor. If the flow is at steady state, what is the mass flow rate of the liquid-vapor ammonia mixture leaving the throttle valve?

02-04. A stream of 300 gpm of cold water and another stream of 400 gpm of hot water enter a mixing chamber. If the chamber operates at steady state, and has only one outlet, what is the water flow rate (gpm) at the discharge?

02-05*. A stream of $0.019\,\mathrm{m}^3/\mathrm{s}$ of cold water and another stream of $0.025\,\mathrm{m}^3/\mathrm{s}$ of hot water enter a mixing chamber. Both inlet pipes have an internal diameter of 77 mm. The chamber operates at steady state, and has only one outlet. It is required that the flow velocity at the discharge must not exceed the flow velocity of the cold water inlet. The smallest allowed internal diameter (mm) for the outlet pipe is most nearly:

(A) 0.117

(B) 77

(C) 117

(D) 234

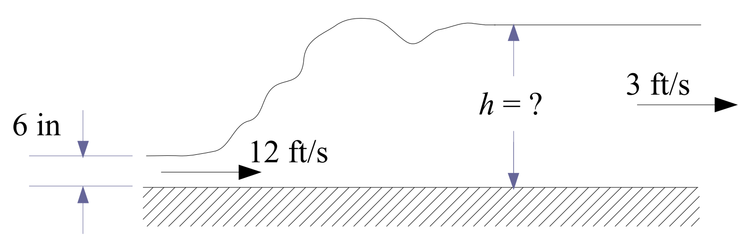
02-06*. A hydraulic jump is in place downstream from a spillway as indicated in the figure depicting a channel of constant width. The depth of the water upstream of the jump is 6 inches and the average stream velocity is 12 ft/s. Just downstream of the jump, the average stream velocity is 3 ft/s. The water depth (ft) just downstream of the jump is most nearly:

(A) 1.5

(B) 2

(C) 24

(D) 48



02-07*. An airplane jet engine receives air at a rate of 70 lbm/s. Simultaneously, fuel enters the engine at a steady rate of 0.55 lbm/s. The average velocity of the exhaust gases is 1,550 ft/s relative to the engine. The engine exhaust effective cross-sectional area is 3.5 ft^2. The density (lbm/ft^3) of the exhaust gases is most nearly:

(A) 0.005

(B) 0.08

(C) 0.010

(D) 0.013

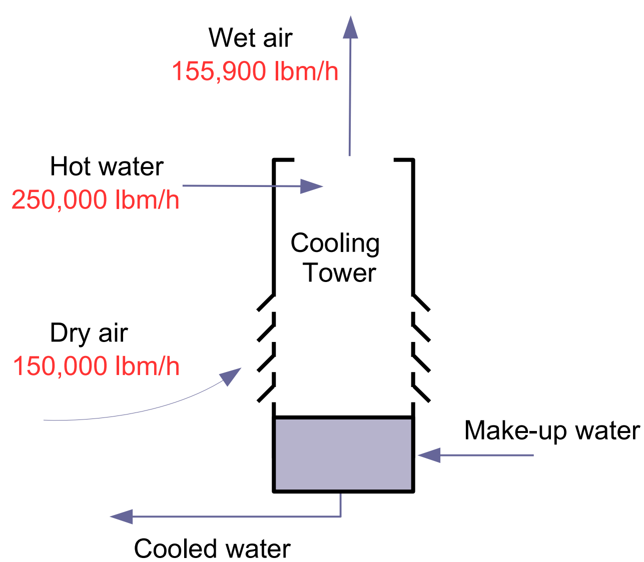
02-08* The evaporative cooling tower shown in the figure is used to cool water (which enters at a rate of 250,000 lbm/h) from 100°F to 80°F. Dry air (no water vapor) flows into the tower at a rate of 150,000 lbm/h. The rate of wet air flow out of the tower is 155,900 lbm/h. Because some of the hot water evaporates, some make-up water has to be added so the flow rate of cooled water leaving the tower equals the rate of hot water entering. The required make-up water flow rate (lbm/h) is most nearly:

(A) 5,900

(B) 155,900

(C) 244,100

(D) Can't be determined



02-09* An above-ground water storage tank has a circular cross section with a diameter of 33 ft and a height of 16 ft. Initially, the tank is 10% liquid full. When a loading operation starts, the water is pumped into the tank at a rate of 1,100 gpm. The high-level alarm is triggered when the liquid level reaches 14 ft. The elapsed time (minutes) between beginning loading and triggering the high-level alarm is most nearly:

(A) 25

(B) 41

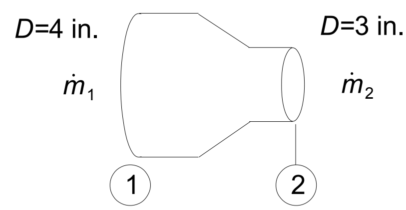
(C) 72

(D) 97

Solutions:

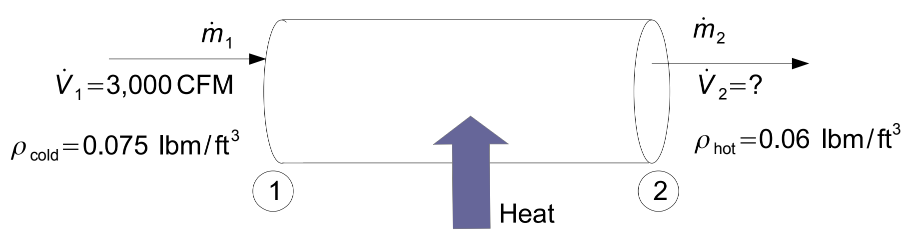
02-01. Perform a mass balance, Equation 2-3:

$$\dot{m}_1 = \dot{m}_2$$
$$\rho A_1 V_1 = \rho A_2 V_2$$
$$\Rightarrow \frac{V_2}{V_1} = \frac{A_1}{A_2} = \left(\frac{D_1}{D_2}\right)^2 = 1.722$$



The percentage increase in velocity is of approximately 72%

02-02.

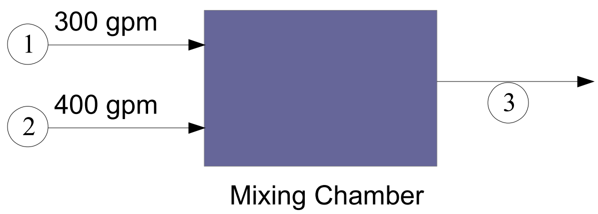


Perform a mass balance, Equation 2-3:

$$\dot{m}_1 = \dot{m}_2$$
$$\rho_1 \dot{V}_1 = \rho_2 \dot{V}_2$$
$$\Rightarrow \dot{V}_2 = \frac{\rho_1}{\rho_2} \dot{V}_1$$
$$= \frac{0.075\,\text{lbm/ft}^3}{0.06\,\text{lbm/ft}^3} \times 3{,}000\,\text{CFM}$$
$$= 3{,}750\,\text{CFM}$$

02-03. If the device is operating at steady state, the mass flow rate downstream and upstream of the device is the same, because of mass conservation. Therefore, the mass flow rate of the liquid vapor mixture leaving the valve is also 8 kg/s

02-04.

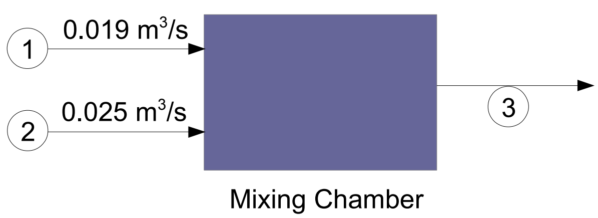


Perform a mass balance, Equation 2-2:

$$\dot{m}_1+\dot{m}_2=\dot{m}_3$$

Since water is modeled as incompressible, we can rewrite the above equation in terms of volumetric flow rates, $\dot{V}_3=\dot{V}_1+\dot{V}_2$. Therefore, $\dot{V}_3=700\,\text{gpm}$.

02-05*. The correct answer is (C). From a mass balance on the mixing chamber, we obtain the volume flow rate at the discharge of $\dot{V}_3=0.019\,\text{m}^3/\text{s}+0.025\,\text{m}^3/\text{s}=0.044\,\text{m}^3/\text{s}$



The flow velocity at section 1, is:

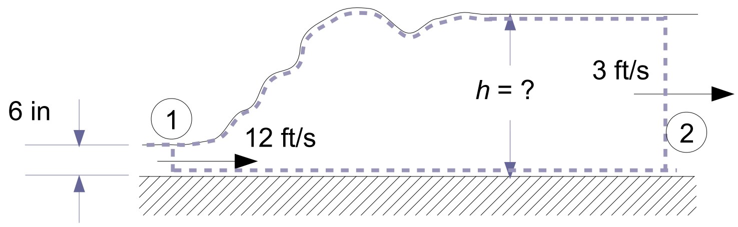
$$V_1=\frac{\dot{V}_1}{A_1}=\frac{0.019\,\frac{\text{m}^3}{\text{s}}}{\frac{\pi}{4}\times\left(77\times10^{-3}\,\text{m}\right)^2}=4.08\,\frac{\text{m}}{\text{s}}$$

Since (per the problem statement) the velocity at 3 has to match this value, then

$$A_3=\frac{\dot{V}_3}{4.08\,\frac{\text{m}}{\text{s}}}=\frac{0.044\,\frac{\text{m}^3}{\text{s}}}{4.08\,\frac{\text{m}}{\text{s}}}=4.08\,\frac{\text{m}}{\text{s}}=0.01078\,\text{m}^2$$

$$\Rightarrow D_3=\sqrt{\frac{4\,A_3}{\pi}}=\sqrt{\frac{4\times0.01078\,\text{m}^2}{\pi}}=0.1172\,\text{m}=117.2\,\text{mm}$$

02-06*. Define a control volume with the dashed lines in the figure:



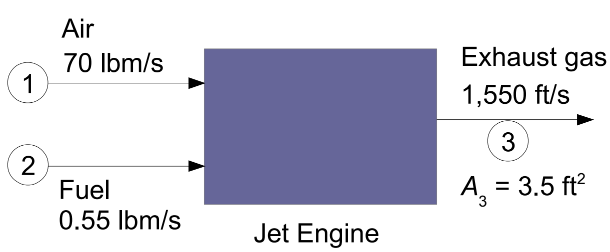
So we have one inlet and one outlet, thus $\dot{m}_1 = \dot{m}_2$, or $A_1 V_1 = A_2 V_2$. If we denote the channel width as W, then:

$$[W \cdot (6\,\text{in})] \times V_1 = [W \cdot h] \times V_2$$

$$\Rightarrow h = 6\,\text{in} \times \frac{V_1}{V_2} = 6\,\text{in} \times \frac{12\,\text{ft/s}}{3\,\text{ft/s}} = 24\,\text{in} \times \left|\frac{1\,\text{ft}}{12\,\text{in}}\right| = 2\,\text{ft}$$

The correct answer is (B)

02-07*. The correct answer is (D)



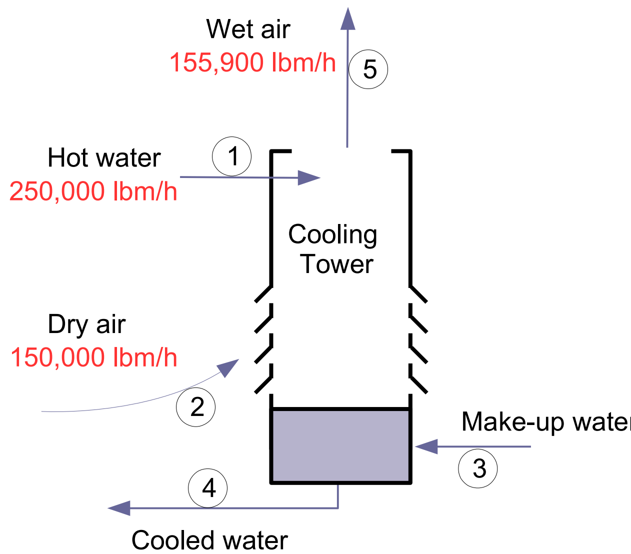
$$\dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

$$\rho_3 A_3 V_3 = \dot{m}_1 + \dot{m}_2$$

$$\rho_3 = \frac{\dot{m}_1 + \dot{m}_2}{A_3 V_3}$$

$$\rho_3 = \frac{70.55\,\dfrac{\text{lbm}}{\text{s}}}{3.5\,\text{ft}^2 \times 1{,}550\,\dfrac{\text{ft}}{\text{s}}} = 0.013\,\frac{\text{lbm}}{\text{ft}^3}$$

02-08*. The correct answer is (A)



$$\sum_i \dot{m}_{in,i} = \sum_j \dot{m}_{out,j}$$

$$\dot{m}_1 + \dot{m}_2 + \dot{m}_3 = \dot{m}_4 + \dot{m}_5$$

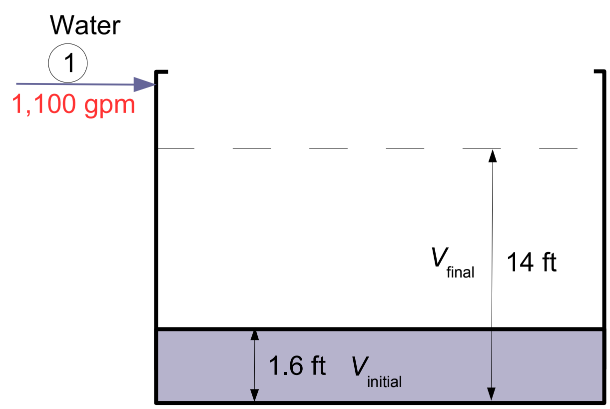
But, from the problem statement, $\dot{m}_1 = \dot{m}_4$,

therefore:

$$\dot{m}_3 = \dot{m}_5 - \dot{m}_2$$

$$\dot{m}_3 = 155{,}900\,\text{lbm/h} - 150{,}000\,\text{lbm/h}$$
$$= 5{,}900\,\text{lbm/h}$$

02-09*. The correct answer is (C).



At the beginning of the filling process, $t = 0$, the tank is 10% liquid full, so the initial volume of water is:

$$V_{\text{initial}} = 0.1 \times \frac{\pi}{4}(33\,\text{ft})^2(16\,\text{ft}) = 1{,}368.5\,\text{ft}^3$$

The volume when the high-level alarm is triggered is:

$$V_{\text{final}} = \frac{\pi}{4}(33\,\text{ft})^2(14\,\text{ft}) = 11{,}974\,\text{ft}^3$$

and the total volume added is:

$$\Delta V = V_{\text{final}} - V_{\text{initial}} = 10{,}605.5\,\text{ft}^3$$

The rate of change of volume in the tank is used by writing Equation 2-1 for an incompressible liquid:

$$\frac{d(\text{VOL})}{dT} = \dot{V}_{\text{in}} \qquad \text{(there is only one inlet and no outlets)}$$

$$\frac{d(\text{VOL})}{dT} = 1{,}100\,\frac{\text{gal}}{\text{min}} \times \left|\frac{1\,\text{ft}^3}{7.48052\,\text{gal}}\right| = 147\,\frac{\text{ft}^3}{\text{min}}$$

At this rate, it takes $10{,}605.5\,\text{ft}^3/(147\,\text{ft}^3/\text{min}) \approx 72\,\text{min}$ to reach the high-level.

PART II: FLUID MECHANICS AND HYDRAULICS

01: Basic Information and Properties of Fluids

Physical quantities require quantitative descriptions when solving engineering problems. Consider the density as one such quantity. It is a measure of the mass contained in a unit volume. However, density is not considered a "fundamental" dimension. Only length, mass, time, temperature plus five more[3] are fundamental. All other quantities can be expressed in terms of fundamental dimensions. For instance, the dimensions of force can be related to the fundamental dimensions of mass, length, and time. To give the dimensions of a quantity a numerical value, a set of units must be selected.

All equations must be **dimensionally homogeneous**. That is, every term in an equation must have the same units. If, at some stage of an analysis, you find yourself in a position to add two quantities that have different units, it is a clear indication that something went wrong at an earlier stage. So checking dimensions can serve as a valuable tool to spot errors. We strongly encourage you to always keep track of units and never, ever, write down a number without its accompanying units. Lack of experience and carelessness with units and unit conversions is one of the biggest hindrances towards success with the P.E. exam.

Unity conversion ratios are identically equal to 1 and are unit-less, and thus such ratios (or their inverses) can be inserted conveniently into any calculation to properly convert units, because any quantity multiplied (or divided) by 1 remains unchanged. For example, the quantity:

$$\left|\frac{1\,\text{lbf}}{\left(32.174\,\dfrac{\text{lbm}\cdot\text{ft}}{\text{s}^2}\right)}\right|$$

is a ratio of two quantities that are identical, so it is equal to 1. Likewise, quantities such as:

$$\left|\frac{12\,\text{in}}{1\,\text{ft}}\right| \qquad \left|\frac{7.48052\,\text{gallons}}{1\,\text{ft}^3}\right| \qquad \left|\frac{6.89476\,\text{kPa}}{1\,\text{psi}}\right|$$

are all identically equal to 1 and are frequently inserted into calculations to ensure the dimensional homogeneity of equations. Some books insert the archaic gravitational constant g_c defined as $g_c = 32.174\,\text{lbm}\cdot\text{ft}/\left(\text{lbf}\cdot\text{s}^2\right)$ into equations in order to force units to match. This practice leads to unnecessary confusion and is strongly discouraged. We recommend you instead use unity conversion

3 The others are electric current, luminous intensity, plane angle, solid angle, and amount of substance.

ratios.

When we say that equations must be dimensionally homogeneous, we mean that the dimensions of the left side of the equation must be the same as those on the right side, and all additive separate terms must have the same dimensions. We accept as a fundamental premise that all equations describing physical phenomena must be dimensionally homogeneous. If this were not true, we would be attempting to equate or add unlike physical quantities, which would not make sense. For example, the equation for the velocity, V , of a uniformly accelerated body is:

$$V = V_0 + at$$

Here we note that the dimensions of all three terms are length/time – thus, the equation is dimensionally homogeneous. To illustrate the use of unity conversion ratios consider this example (formatted in "PE style"):

EXAMPLE. The velocity of a uniformly accelerated body is given by $V = V_0 + at$. Therefore, the velocity (inches per second) at $t = 0.25\,\text{min}$ if the initial velocity $V_0 = 50\,\text{ft/min}$ and $a = 0.85\,\text{ft/s}^2$ is most nearly:

(A) 50.85
(B) 163
(C) 815
(D) 9780

The first way the careless test-taker messes this one up is by not writing down the units and doing the following:

$$V = V_0 + at = 50 + 0.85 \times 1 = 50.85$$

This is of course terribly wrong, but people actually do this! As you can see, 50.85 is one of the answer choices. So, careless test-taker #1 would select (A) 50.85, and be wrong. The actual test will have "distractor" answers like this. Be careful.

Careless test taker #2 does a little bit better, and actually gets a "correct" answer as follows:

$$V = V_0 + at = 50\,\frac{\text{ft}}{\text{min}} + 0.85\,\frac{\text{ft}}{\text{s}^2} \times 0.25\,\text{min}$$
$$= 50\,\frac{\text{ft}}{\text{min}} + 0.85\,\frac{\text{ft}}{\text{s}^2} \times \left|\frac{60\,\text{s}}{1\,\text{min}}\right|^2 \times 0.25\,\text{min} = 50\,\frac{\text{ft}}{\text{min}} + 765\,\frac{\text{ft}}{\text{min}} = 815\,\frac{\text{ft}}{\text{min}}$$

Note the use of the unity conversion factor on the second term, to ensure dimensional homogeneity of the calculation. Note that 815 is one of the answer choices. However, the question specifies the units of the answer must be in inches per second. Choosing (C) 815, would be wrong – even though the answer is correct! We leave it as an exercise for you to confirm that the correct answer is (B).

Some valid equations contain constants with dimensions. For example, the equation for the distance, d, traveled by a body in free-fall can be written as:

$$d = 16.1\, t^2$$

and a check of the dimensions reveals that the constant 16.1 must have the dimensions of length over time squared, if the equation is to be dimensionally homogeneous. This equation is actually a particular case of the more general equation from physics for freely falling bodies:

$$d = g \frac{t^2}{2}$$

in which g is the acceleration of gravity. This equation is dimensionally homogeneous and valid in any system of units, but for For $g = 32.2\,\text{ft/s}^2$ the equation reduces to $d = 16.1\, t^2$ which is valid only for the system of units using feet and seconds.

Pressure results from compressive forces acting on an area of a continuous medium. Pressure is a scalar function; it acts equally in all directions at a given point for a fluid whether it is quiescent or in motion. The **absolute pressure** reaches zero when an ideal vacuum is achieved (i.e., in a space where there are no molecules) thus negative absolute pressures are not possible. In addition to the absolute pressure scale, pressures can be measured with respect to the local atmospheric pressure. The term **gage-pressure** refers to values of pressure measured relative to the local atmospheric value. The gage-pressure is negative whenever the absolute pressure is lower than the local atmospheric pressure. Negative gage-pressure values are referred to as **vacuum** pressures. The following equation provides the conversion from gage to absolute pressures:

$$p_{\text{absolute}} = p_{\text{gage}} + p_{\text{atm}} \tag{1-1}$$

In the International System of Units (SI), pressure is measured in **Pascals** (Pa) with the SI prefixes kilo-, Mega-, and Giga-, being among the most used in ordinary engineering applications.

1 kPa = 10^3 Pa
1 MPa = 10^3 kPa = 10^6 Pa
1 GPa = 10^3 MPa = 10^6 kPa = 10^9 Pa

The **bar** is a unit of pressure of the "metric system", but is not formally approved as part of the SI. It is defined as 100 kPa, or 0.1 MPa. In the US Customary System (USCS) of units, pressure is typically expressed in pounds-force per square inch (lbf/in^2, or **psi**) or pounds-force per square foot (**psf**).

In general, one of the words "absolute" or "gage" typically follows a value if the given pressure is given as an absolute or gage pressure, respectively (e.g., " $p=750\,\text{kPa}$ absolute") but sometimes it is specified in the units themselves, especially in the USCS. For example $p=580\,\text{psia}$ refers to an absolute pressure of 580 pounds-force per square inch, and " $p=3.5\,\text{psig}$ " refers to a gage pressure of 3.5 pounds force per square inch.

The local atmospheric pressure varies with elevation, and the "standard" value at sea level is defined as 101.3 kPa, 14.7 psi, 30 in. Hg, 760 mmHg, or 1.013 bar.

A fluid's **density**, ρ ,is defined as the mass of a unit of volume, and the **specific volume**, γ ,is defined as the weight of a unit of volume:

$$\gamma=\rho\, g \tag{1-1}$$

where g is the local gravity. The units of specific weight are N/m^3 and lbf/ft^3. The specific gravity SG is often used to determine a fluid's density or specific weight. It is defined as the ratio of the density of a substance to that of water at a reference temperature:

$$SG=\frac{\rho}{\rho_{\text{water, std}}}=\frac{\gamma}{\gamma_{\text{water, std}}} \tag{1-2}$$

If the reference temperature is the typically used value of 4°C then:

$$\rho_{\text{water, std}}=1000\,\frac{\text{kg}}{\text{m}^3}=62.4\,\frac{\text{lbm}}{\text{ft}^3}=0.0361\,\frac{\text{lbm}}{\text{in}^3} \qquad \gamma_{\text{water, std}}=9810\,\frac{\text{N}}{\text{m}^3}=62.4\,\frac{\text{lbf}}{\text{ft}^3}=0.0361\,\frac{\text{lbf}}{\text{in}^3}$$

For fluids at rest, it can be shown that the pressure does not vary in the horizontal plane (which is defined as the plane normal to the gravity vector), and if we assume the density is constant then:

$$\frac{p}{\gamma}+z=\text{constant} \tag{1-3}$$

where z is positive in the vertically upward direction, so that pressure increases with depth. The quantity $(p/\gamma)+z$ is known as the **piezometric head**. If we define as $z=0$ the location of an interface between the liquid and air above it (this is known as a free surface) at atmospheric pressure, then $p=0$ (gage) at $z=0$, thus:

$$p=\gamma\, h \tag{1-4}$$

where h is the depth below the free surface. Equation 1-4 is useful in converting pressure to an equivalent height of liquid. For example, atmospheric pressure is often expressed in millimeters of mercury – this means that the atmospheric pressure is equal to the pressure at a certain depth in a mercury column, and by knowing the specific weight of mercury, we can determine the depth using Equation 1-4.

Manometers are instruments that use columns of liquid to measure pressures. Figure 1-1 shows a U-tube manometer, which is used to measure relatively small pressures. In the figure, point "1" is located in the center of a pipe that extends in the direction perpendicular to the page, and point "2" is at the free surface of the right column, exposed to atmosphere.

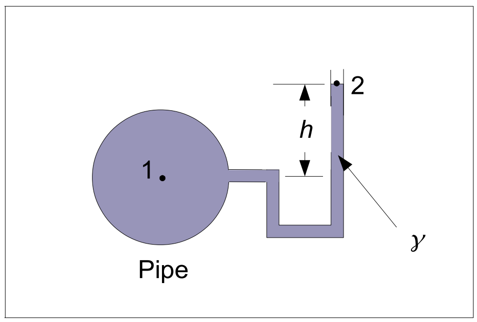


Figure 1-1: U-tube Manometer.

We can use Equation 1-3 to write:

$$p_1 + \gamma z_1 = p_2 + \gamma z_2$$

where the datum (i.e., the location where $z=0$) can be located anywhere we want. We know also that $p_2 = 0$ psig (we could also use $p_2 = p_{\text{atm}}$ if we wish to work with absolute pressures). Choosing now $z_1 = 0$, the above expression yields: $p_1 = \gamma h$.

Figure 1-2 shows a manometer that could be used to measure larger pressures since we could choose a fluid with a large specific weight, γ_B (for example if fluid 1 is water and fluid 2 – the red one – is mercury, then $\gamma_B = 13.6\, \gamma_A$). The pressure at location 1 can be determined by defining 4 points as shown in Figure 1-2.

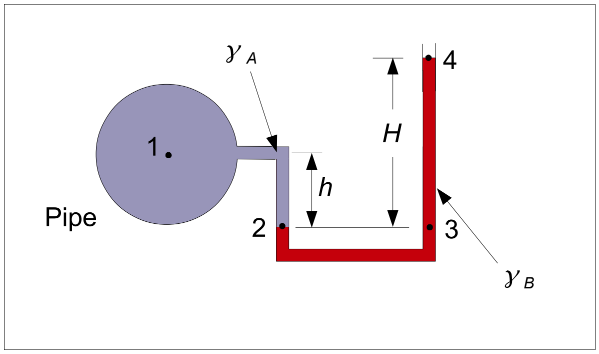


Figure 1-2: U-tube Manometer with dedicated manometer fluid

We can use Equation 1-3 to write:

$$p_1 + \gamma_A z_1 = p_2 + \gamma_A z_2$$

$$p_3 + \gamma_B z_3 = p_4 + \gamma_B z_4$$

but, $p_2 = p_3$ because points 2 and 3 are on the same horizontal plane ($z_2 = z_3$) and within the same fluid. We can add the above equations to obtain:

$$p_1 + \gamma_A (z_1 - z_2) = \gamma_B (z_4 - z_3) + p_4$$

and we can set $p_4 = 0$ and work with gage pressures. The resulting expression for p_1 is:

$$p_1 = \gamma_B H - \gamma_A h$$

Note that if fluid A (the one in the pipe) is a gas, and fluid B is a liquid, it generally follows that $\gamma_A \ll \gamma_B$ and the second term in the above expression can be safely neglected.

The lay person's understanding of viscosity is that it is related to the internal stickiness of a fluid. In more technical terms, **viscosity** is linked to the rate of deformation of a fluid under shear stress. For a given applied shear stress, a high viscosity fluid will experience smaller deformation than a low viscosity fluid under the same stress.

Consider the fluid within a small gap between two concentric cylinders, as shown in Figure 1-3. An externally applied torque causes the rotation of the inner cylinder, while the outer cylinder remains stationary. The resistance to the rotation is due to the fluid's viscosity.

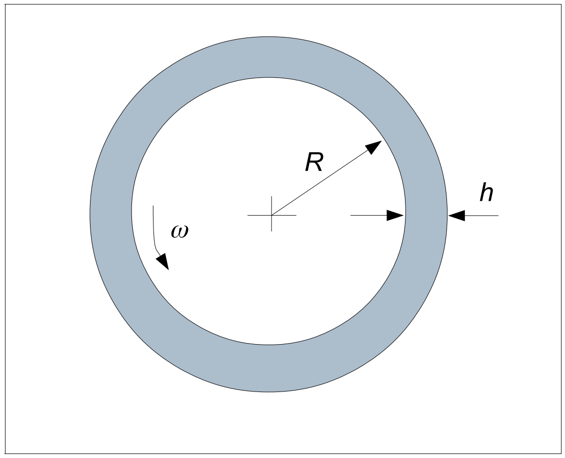


Figure 1-3: Fluid being sheared between two cylinders: Fixed outer cylinder and rotating inner cylinder.

Experimental observations confirm that the fluid sticks to both cylinders, which is known as the **no-slip condition**. The fluid layer adjacent to the inner cylinder will have a velocity equal to the tangential velocity of the cylinder, ωR, where ω is the rotational speed of the inner cylinder (in rad/s) and R is the radius of the inner cylinder. The fluid layer adjacent to the outer (stationary) cylinder will be stationary. The fluid velocity u in the thin gap varies linearly from zero to ωR [4]. If the gap between the cylinders is h, then the velocity gradient du/dr across the gap is $(\omega R)/h$.

When a torque T is applied to the inner cylinder, the fluid in the gap experiences a shear stress τ, which is related to the velocity gradient:

$$\tau = \mu \left| \frac{du}{dr} \right| = \mu \frac{(\omega R)}{h} \tag{1-4}$$

Where the constant of proportionality between sheer stress and velocity gradient (in this very simple flow) is the viscosity, μ. It can be shown that the torque (and thus the power) required to rotate the inner cylinder at a given rate is linearly proportional to the viscosity. Since the torque depends on the viscosity, this type of cylinder gap arrangement is used as a **viscometer** – a device used to measure viscosity.

A shear stress can also be induced in a controlled manner for a fluid sandwiched within a flat plate arrangement such as in Figure 1-4.

4 If the gap width h is not small relative to R the velocity variation will not be linear.

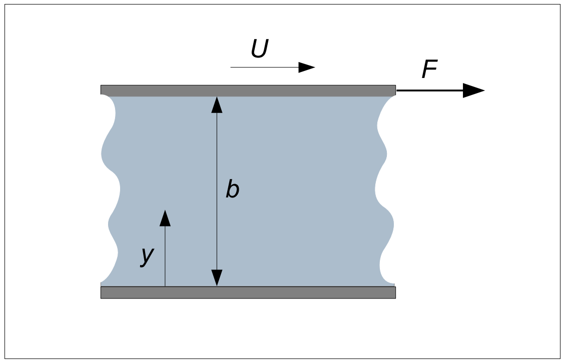


Figure 1-4: Fluid being sheared between two flat plates: Fixed lower plate and moving upper plate.

The force F being applied at the top plate causes the upper plate to travel at a constant velocity U. Because of the no-slip condition, the fluid adjacent to the bottom plate is stationary and the fluid adjacent to the top plate will travel with the velocity U. Therefore the fluid velocity u depends on the y coordinate so that $u(0)=0$ and $u(b)=U$ where b is the gap width. The velocity gradient within the gap is constant and equal to U/b, hence the shear stress in any layer within the fluid-filled gap is:

$$\tau = \mu \left|\frac{du}{dy}\right| = \mu \frac{U}{b} \tag{1-5}$$

If the plate surface area is A then the shear stress $\tau = F/A$.

If the shear stress in a fluid is directly proportional to the velocity gradient (shear strain rate), the fluid is said to be a **Newtonian fluid**. If the shear stress varies in a non-linear way with the strain rate, then the fluid is **non-Newtonian** (liquid plastics, blood, slurries, toothpaste, etc) and depending on the functional form of this relationship the fluids are further classified as thixotropic, dilatant, Bingham plastics, rheopectic, etc. **Rheology** is the branch of physics that deals with this study of deformation and flow of materials.

From $\tau = \mu\,(du/dy)$ it can be readily deduced that the dimensions of viscosity are force×time/length^2. Thus, in the USCS of units viscosity is given as lbf·s/ft^2 and in SI units as N·s/m^2. The poise (P) is an alternatively used viscosity unit and 1 P = 0.1 N·s/m^2. The poise is often used with the prefix "centi-" because the viscosity of water at 20 °C is almost exactly 1 cP where 1 cP = 10^{-3} N·s/m^2. Another common unit for viscosity is the "reyn" and 1 reyn = 1lbf·s/in^2.

The group μ/ρ appears often in the derivation of equations, so it has become customary to give it a name, **kinematic viscosity**:

$$\nu = \frac{\mu}{\rho} \tag{1-6}$$

and thus the viscosity μ is also referred to as “dynamic” viscosity. The dimensions of kinematic viscosity are length^2/time, and the USCS units are ft^2/s and SI units are m^2/s. The stoke[5] (St) is an alternatively used unit of kinematic viscosity and 1 St = 10^{-4} m^2/s. The stoke is often used with the prefix “centi-” because the kinematic viscosity of water at 20 °C is almost exactly 1 cSt (10^{-6} m^2/s).

Given a pressure change Δp applied to a fluid, a packet of fluid with volume $\forall$ will experience a change in relative volume $(\Delta \forall)/\forall$. The **bulk modulus of elasticity**, K gives the relative change in volume for a given change in pressure applied to a fluid at constant temperature, T :

$$K = \left.\frac{\Delta p}{(\Delta \forall)/\forall}\right|_T = \rho \left.\frac{\Delta p}{\Delta \rho}\right|_T \tag{1-7}$$

For gases, it can be shown that the bulk modulus is equal to the pressure. For liquid water at standard conditions, the bulk modulus is roughly 310,000 psi – this means that to cause a 1% change in density (i.e., $(\Delta \rho)/\rho = 0.01$) of water, a pressure rise of 3,100 psi is required. The compressibility, β is simply the reciprocal of the bulk modulus of elasticity $\beta = 1/K$.

The **buoyant force** is the resultant force acting on a body that is completely submerged in a fluid, or floating so that it is only partially submerged. Since pressure increases with depth, the pressure forces acting from below are larger than the pressure forces acting from above. Therefore, a net upward vertical force results.

The buoyant force is given by:

$$F_B = \gamma \forall \tag{1-8}$$

where γ is the specific weight of the fluid and $\forall$ is the volume of the fluid displaced by the body. Thus, the buoyant force has a magnitude equal to the weight of the fluid displaced by the body, and is directed vertically upward. This result is commonly referred to as **Archimedes’ principle**. The buoyant force passes through the centroid of the displaced volume, and the point through which the buoyant force acts is called the **center of buoyancy**.

5 In the United States, it is more common to use the singular “Stoke”, but the unit is named after George G Stokes, hence in the rest of the world it is more common to hear “Stokes” as a unit of kinematic viscosity.

These same results apply to floating bodies that are only partially submerged, if the specific weight of the fluid above the liquid surface is very small compared with the liquid in which the body floats. Since the fluid above the surface is usually air, for such applications this condition is satisfied. For floating or partially submerged bodies, be sure to remember that $\forall$ in Equation 1-8 is the submerged volume.

The equations in this section can be found at the beginning of Chapter 3 in the PEMH, with the exception of equations for manometers, which can be located by searching for the phrase "manometers" in the PDF.

PROBLEMS:

01-01. If V is a velocity, determine the dimensions of Z , α , and G , which appear in the dimensionally homogeneous equation $V = Z(\alpha - 1) + G$

01-02. The pressure difference, Δp , across a partial blockage in an artery is approximated by the equation:

$$\Delta p = K_v \frac{\mu V}{D} + K_u \left(\frac{A_0}{A_1} - 1 \right)^2 \rho V^2$$

where V is the blood velocity, μ the blood viscosity, ρ the blood density, D the artery diameter, A_0 the area of the unobstructed artery, and A_1 the area of the blockage. Determine the SI dimensions of the constants K_v and K_u .

01-03. A pressure of 4 psig is measured at an elevation of 6,560 ft – where the atmospheric pressure is 11.5 psia. What is the absolute pressure in (a) kPa, (b) mm of Hg, and (c) ft of water?

01-04. A gage reads a vacuum of 24 kPa. What is the absolute pressure (a) at sea level, and (b) at an altitude of 4000 m where the atmospheric pressure is 0.616 bar?

01-05. Calculate the gage pressure (in psi) in a pipe transporting air if a U-tube manometer measures 9.85 inches of mercury. Note that the weight of air in the manometer is negligible.

01-06. If the air pressure in a pipe is 65 psig, what will a U-tube manometer with mercury indicate?

01-07*. The Weber number is a dimensionless parameter, given by:

$$\mathrm{We}=\frac{\rho V^2 L}{\sigma}$$

where ρ is density, V is a velocity, L is a length, and σ is a material property of the fluid. Since the Weber number is dimensionless, the units of σ must be:

(A) lbm/(ft·s)

(B) lbf/ft

(C) ft/lbf

(D) s^2/lbm

01-08*. A formula to estimate the volume rate of flow, Q, flowing over a dam of length, B, is given by the equation $Q=3.09\,B\,H^{3/2}$,where H is the depth of the water above the top of the dam. This formula gives Q in ft^3/s when B and H are in feet. An equivalent formula that gives Q in gallons per day when B and H are in feet is:

(A) $Q=0.00027\,B\,H^{3/2}$

(B) $Q=0.413\,B\,H^{3/2}$

(C) $Q=36{,}000\,B\,H^{3/2}$

(D) $Q=2\times 10^6\,B\,H^{3/2}$

01-09*. While performing a calculation, an engineer arrives at the following equation to calculate a force: $F=\dot{m}V+pA$, where $\dot{m}=600\,\mathrm{kg/min}$, $V=2\,\mathrm{m/s}$, $p=29.7\,\mathrm{kPa}$, and $A=50.3\,\mathrm{cm}^2$.

Under these conditions, the force F (N) is most nearly:

(A) 35

(B) 169

(C) 1513

(D) 2694

01-10*. While performing a calculation, an engineer arrives at the following equation to calculate a volumetric flow rate: $Q = F/(\rho V)$,where $F = 35$ pounds-force, $\rho = 6.26\,\text{lbm}/\text{gallon}$, and $V = 15\,\text{ft}/\text{s}$. Under these conditions, the flow rate Q (cubic feet per hour) is most nearly:

(A) 0.373

(B) 12

(C) 5,775

(D) 43,172

01-11*. A heat transfer oil with a specific gravity of 0.86 flows through a pipe. The reading from a U-tube manometer is 9.5 in Hg. The oil in the manometer is depressed 5 inches below the pipe centerline, and the local atmospheric pressure is 14.7 psi. Under these conditions, the absolute pressure (psi) of the oil in the pipe at the location of the manometer is most nearly:

(A) 4.5

(B) 7.8

(C) 14.7

(D) 19.2

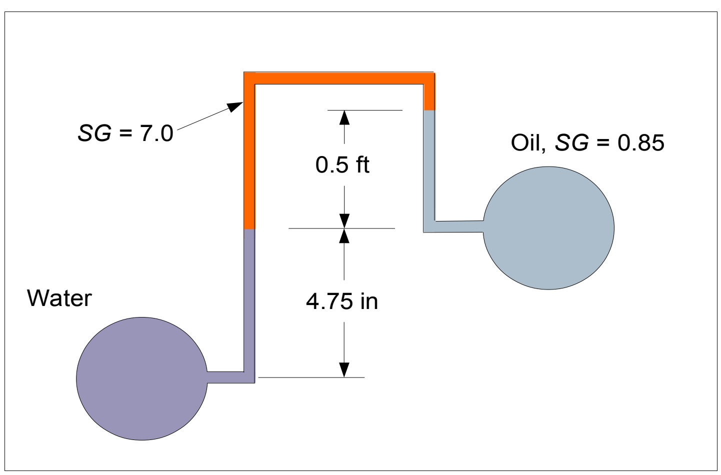
01-12*. The pressure in the water pipe is 2.5 psig. The pressure (psig) in the oil pipe is most nearly:

(A) 1.0

(B) 2.0

(C) 2.2

(D) 4.5



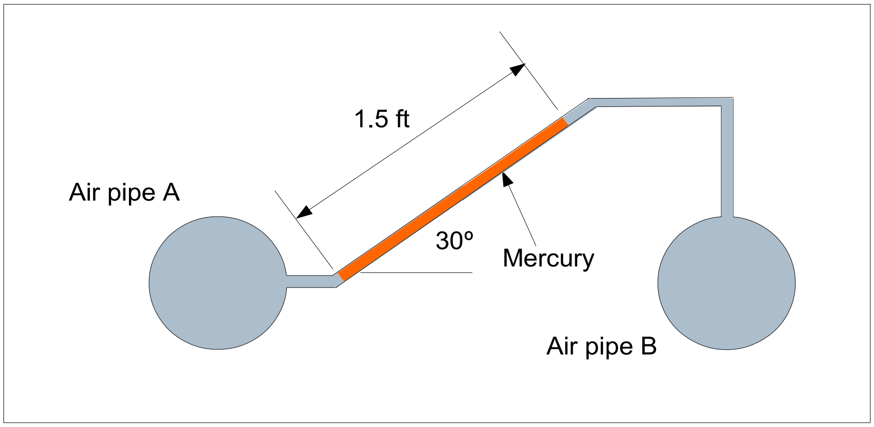
01-13*. The pressure in air pipe A is 15 psig. The pressure (psig) in air pipe B is most nearly:

(A) 6.2

(B) 10.6

(C) 14.6

(D) 15.0



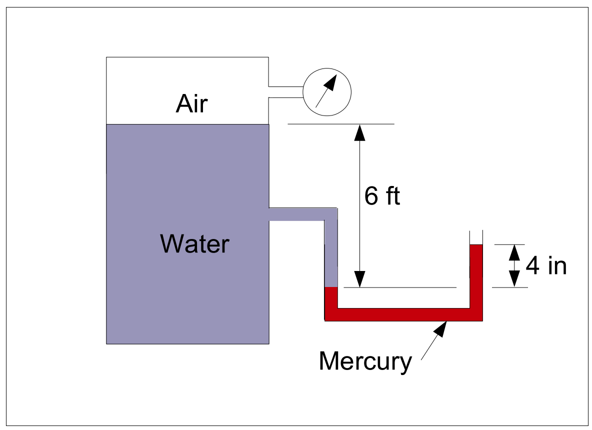
01-14*. The reading of the pressure gage in psi is most nearly:

(A) a vacuum of 2.5

(B) a vacuum of 1.6

(C) a vacuum of 0.64

(D) 1.74



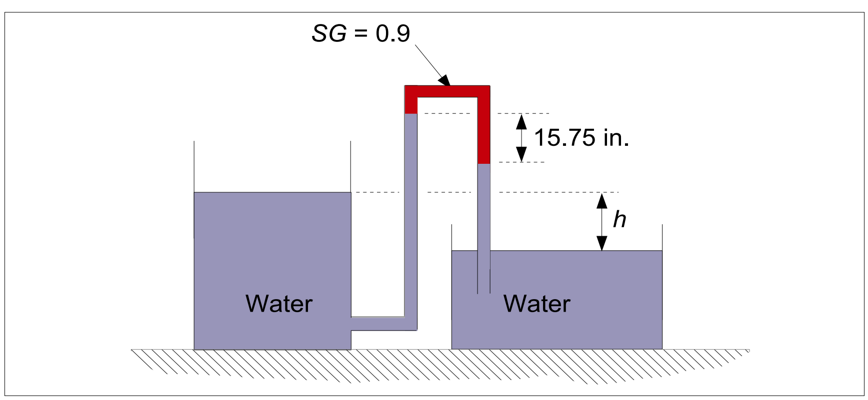
01-15*. The elevation difference *h* (ft) between the water levels in the two open tanks is most nearly:

(A) 0.13

(B) 1.6

(C) 2.6

(D) 3.5



01-16*. Crude oil with a viscosity of 6.6×10^{-4} reyn is contained between parallel plates. The bottom plate is fixed and the upper plate moves when a force F is applied. The distance between the two plates is 0.1 inches and the effective area of the upper plate is 1.4 ft^2. The force (pounds-force) required to translate the plate with a velocity of 3 ft/s is most nearly:

(A) 0.03

(B) 0.33

(C) 47.9

(D) 95.8

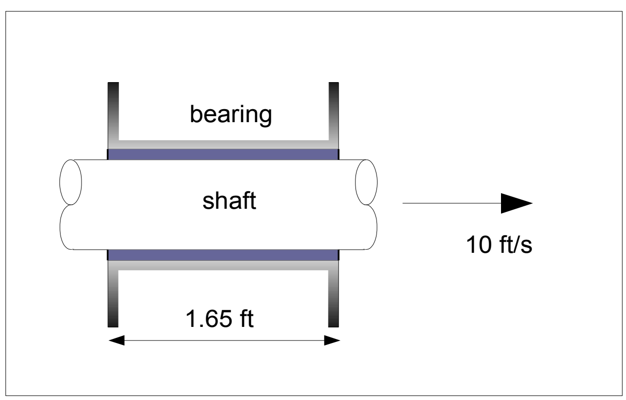
01-17*. A 1 inch-diameter shaft is pulled through a cylindrical bearing as shown in the figure. The lubricant that fills the 0.012-inch gap between the shaft and bearing is an oil having a kinematic viscosity of 0.0086 ft^2/s and a specific gravity of 0.91. The force required to pull the shaft at a velocity of 10 ft/s is most nearly:

(A) 4.56

(B) 23.5

(C) 30.8

(D) 65.6



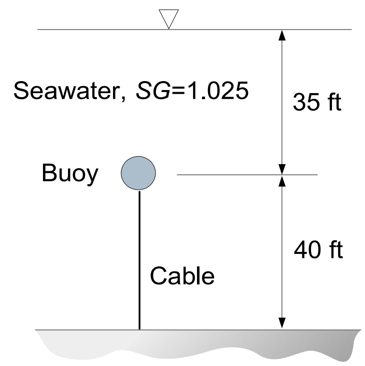
01-18*. A spherical, hollow, metal buoy with a diameter of 60 inches, weighing 1,900 lbf is anchored to the sea floor with a cable as shown in the figure. Under these conditions, the tension (pounds-force) in the cable is most nearly:

(A) 1,900

(B) 2,300

(C) 4,200

(D) 6,500



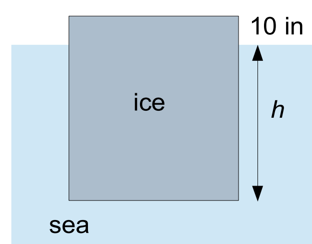
01-19*. A large cubic ice block $SG = 0.92$ floats in seawater $SG = 1.025$. A 10-inch high portion of the ice block extends above the surface of water. The height (ft) of the ice block below the surface is most nearly:

(A) 68

(B) 78

(C) 88

(D) 98



Solutions:

01-01. The term G must have the same dimensions as V, hence G must have dimensions of length/time. This is sometimes expressed by using a bracket operator to mean "the units of": $[G] = \text{ft/s}$. Also, the parameter α must be dimensionless (because it must have the same dimensions as the constant "1"). Therefore $[Z] = \text{ft/s}$.

01-02. Each term in the equation has units of pressure, therefore:

$$\left[K_u\left(\frac{A_0}{A_1}-1\right)^2\rho V^2\right] = \text{Pa} = \frac{\text{N}}{\text{m}^2} = \frac{\text{kg}}{\text{m}\cdot\text{s}^2}$$

where the bracket operator means "the units of". Since the units of a product of terms equals the product of the units of each of the terms:

$$[K_u]\left[\left(\frac{A_0}{A_1}-1\right)^2\right][\rho]\left[V^2\right] = \frac{\text{kg}}{\text{m}\cdot\text{s}^2}$$

since, the area ratio is dimensionless, we have:

$$\left[\left(A_0/A_1-1\right)^2\right] = 1$$

Therefore

$$\begin{aligned}
&[K_u][\rho]\left[V^2\right] = \frac{\text{kg}}{\text{m}\cdot\text{s}^2} \\
&\Rightarrow [K_u]\cdot\frac{\text{kg}}{\text{m}^3}\cdot\frac{\text{m}^2}{\text{s}^2} = \frac{\text{kg}}{\text{m}\cdot\text{s}^2} \\
&\Rightarrow [K_u] = 1
\end{aligned}$$

Therefore, the constant K_u is dimensionless. Similarly:

$$\begin{aligned}
&\frac{[K_v][\mu][V]}{[D]} = \frac{\text{kg}}{\text{m}\cdot\text{s}^2} \\
&\Rightarrow \frac{[K_v]\cdot\dfrac{\text{kg}}{\text{m}\cdot\text{s}}\cdot\dfrac{\text{m}}{\text{s}}}{\text{m}} = \frac{\text{kg}}{\text{m}\cdot\text{s}^2} \\
&\Rightarrow [K_v] = 1
\end{aligned}$$

Therefore, the constant K_v is also dimensionless.

01-03. Use Equation 1-1:

$$p_{\text{absolute}} = p_{\text{gage}} + p_{\text{atm}} = 4\,\frac{\text{lbf}}{\text{in}^2} + 11.5\,\frac{\text{lbf}}{\text{in}^2} = 15.5\text{ psia}$$

(a) $p = 15.5\text{ psi} \times \left|\frac{6.89476\text{ kPa}}{1\text{ psi}}\right| = 106.9\text{ kPa}$

(b) Use Equation 1-4:

$$h = \frac{p}{\gamma_{\text{Hg}}} = \frac{p}{SG_{\text{Hg}}\,\gamma_{\text{H}_2\text{O}}} = \frac{15.5\,\frac{\text{lbf}}{\text{in}^2}}{13.6 \times 0.0361\,\frac{\text{lbf}}{\text{in}^3}} = 31.6\text{ in Hg} \times \left|\frac{25.4\text{ mm}}{1\text{ in}}\right| = 802\text{ mm Hg}$$

(c) Use Equation 1-4:

$$h = \frac{p}{\gamma_{\text{H}_2\text{O}}} = \frac{15.5\,\frac{\text{lbf}}{\text{in}^2}}{0.0361\,\frac{\text{lbf}}{\text{in}^3}} = 429.4\text{ in H}_2\text{O} \times \left|\frac{1\text{ ft}}{12\text{ in}}\right| = 35.8\text{ ft H}_2\text{O}$$

01-04.

a) Use Equation 1-1:

$$p_{\text{absolute}} = p_{\text{gage}} + p_{\text{atm}} = -24\text{ kPa} + 101.3\text{ kPa} = 77.3\text{ kPa}$$

b) Use Equation 1-1:

$$p_{\text{absolute}} = p_{\text{gage}} + p_{\text{atm}} = -24\text{ kPa} + 0.616\text{ bar} \times \left|\frac{100\text{ kPa}}{1\text{ bar}}\right| = 37.6\text{ kPa}$$

01-05. Refer to Figure 1-2. We obtained the expression: $p_1 = \gamma_B H - \gamma_A h$ for the pressure in the pipe. For mercury and air, $\gamma_B \gg \gamma_A$, hence

$$p_1 = \gamma_B H = SG_{\text{Hg}}\,\gamma_{\text{H}_2\text{O}} H = 13.6 \times 0.0361\,\frac{\text{lbf}}{\text{in}^3} \times 9.85\text{ in} = 4.84\text{ psi}$$

01-06. Refer to Figure 1-2. We obtained the expression: $p_1 = \gamma_B H - \gamma_A h$ for the pressure in the pipe. For mercury and air, $\gamma_B \gg \gamma_A$, hence

$$H = \frac{p_1}{\gamma_{\text{Hg}}} = \frac{65\,\frac{\text{lbf}}{\text{in}^2}}{13.6 \times 0.0361\,\frac{\text{lbf}}{\text{in}^3}} = 132.4\ \text{in}$$

01-07*. The correct answer is (B)

For the ratio to be dimensionless, the units of the numerator must be the same as the units of the denominator. We use the bracket operator to mean "the units of":

$$\begin{aligned}[\sigma] &= [\rho][V^2][L] \\ &= \frac{\text{lbm}}{\text{ft}^3} \cdot \frac{\text{ft}^2}{\text{s}^2} \cdot \text{ft} = \frac{\text{lbm}}{\text{s}^2}\end{aligned}$$

This is not any of the answer choices, but at least we can rule out (A) and (D). Choices (B) and (C) involve a unit of force, so we have to express lbm/s^2 with units of force instead of mass.

$$[\sigma] = \frac{[mass]}{\text{s}^2} = \frac{[force]}{[acceleration]\,\text{s}^2} = \frac{\text{lbf}}{\frac{\text{ft}}{\text{s}^2} \cdot \text{s}^2} = \frac{\text{lbf}}{\text{ft}}$$

01-08*. The correct answer is (D)

If the product $3.09\, B\, H^{3/2}$ is in ft^3/s then we multiply it by the appropriate unity conversion factors:

$$3.09\, B\, H^{3/2} \frac{\text{ft}^3}{\text{s}} \times \left| \frac{7.48052\ \text{gal}}{1\ \text{ft}^3} \right| \times \left| \frac{86{,}400\ \text{s}}{1\ \text{day}} \right| = 1{,}997{,}119\, B\, H^{3/2}$$

01-09*. The correct answer is (B)

Plug in the numbers and keep track of units:

$$\begin{aligned}F &= \dot{m} V + p A \\ &= 600\,\frac{\text{kg}}{\text{min}} \times \left| \frac{1\ \text{min}}{60\ \text{s}} \right| \times 2\,\frac{\text{m}}{\text{s}} \ + \ 29.7 \times 10^3\,\frac{\text{N}}{\text{m}^2} \times 50.3\ \text{cm}^2 \times \left| \frac{1\ \text{m}}{100\ \text{cm}} \right|^2 \\ &= 20\ \text{N} \ + \ 149.4\ \text{N} \\ &= 169.4\ \text{N}\end{aligned}$$

01-10*. Plug in the numbers and keep track of units:

$$Q = \frac{F}{\rho V}$$

$$= \frac{35\,\text{lbf} \times \left|\dfrac{32.2\,\text{lbm ft/s}^2}{1\,\text{lbf}}\right|}{6.26\,\dfrac{\text{lbm}}{\text{gal}} \times 15\,\dfrac{\text{ft}}{\text{s}}} \;=\; 12\,\frac{\text{gal}}{\text{s}} \times \left|\frac{3{,}600\,\text{s}}{1\,\text{h}}\right| \times \left|\frac{1\,\text{ft}^3}{7.48052\,\text{gal}}\right| \;=\; 5{,}775\,\frac{\text{ft}^3}{\text{h}}$$

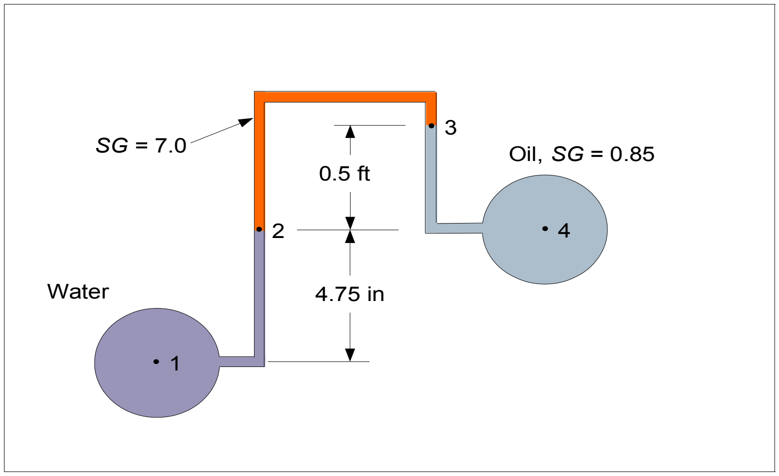
01-11*. The correct answer is (D). Refer to Figure 1-2. We obtained the expression: $p_1 = \gamma_B H - \gamma_A h$ for the pressure in the pipe. In this case, $\gamma_A = 0.86\,\gamma_{H_2O}$, $\gamma_B = 13.6\,\gamma_{H_2O}$, $H = 9.5\,\text{in}$, and $h = 5\,\text{in}$. Therefore,

$$p_1 = \gamma_B H - \gamma_A h = \gamma_{H_2O}(13.6\,H - 0.86\,h) = 0.0361\,\frac{\text{lbf}}{\text{in}^3}(13.6 \times 9.5 - 0.86 \times 5)\,\text{in} = 4.5\,\frac{\text{lbf}}{\text{in}^2} = 4.5\,\text{psi}$$

But this is the gage pressure. We need absolute pressure, therefore: $p_1 = 4.5 + 14.7 = 19.2\,\text{psia}$.

01-12*. The correct answer is (A)

Define points 1 through 4 on the figure.



$$p_1 + \gamma_w z_1 = p_2 + \gamma_w z_2 \;\Rightarrow\; p_2 = p_1 - \gamma_w (z_2 - z_1) \tag{1}$$

Similarly, we can obtain the following:

$$p_3 = p_2 - 7.0\,\gamma_w (z_3 - z_2) \qquad p_4 = p_3 + 0.85\,\gamma_w (z_3 - z_4)$$

We can combine these last 2 expressions (eliminating p_3) to find:

$$p_4 = p_2 - 7.0\,\gamma_w (z_3 - z_2) + 0.85\,\gamma_w (z_3 - z_4)$$

which can be combined with Equation (1) to eliminate p_2 :

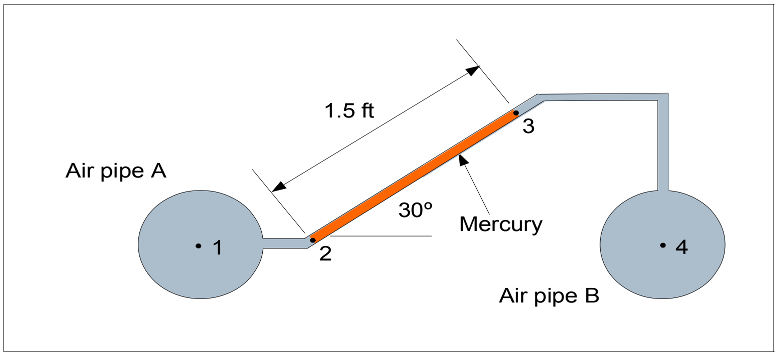
$$\begin{aligned} p_4 &= p_1 - \gamma_w (z_2 - z_1) - 7.0\,\gamma_w (z_3 - z_2) + 0.85\,\gamma_w (z_3 - z_4) \\ &= p_1 - \gamma_w \big((z_2 - z_1) + 7.0 (z_3 - z_2) - 0.85 (z_3 - z_4) \big) \end{aligned}$$

Now insert the given values:

$$p_4 = 2.5 \frac{\text{lbf}}{\text{in}^2} - 0.0361 \frac{\text{lbf}}{\text{in}^3} \left(4.75\,\text{in} + 7.0 \times 0.5\,\text{ft} \times \left| \frac{12\,\text{in}}{1\,\text{ft}} \right| - 0.85 \times 0.5\,\text{ft} \times \left| \frac{12\,\text{in}}{1\,\text{ft}} \right| \right) = 0.996\,\text{psi}$$

01-13*. The correct answer is (B)

Define points 1 through 4 on the figure.

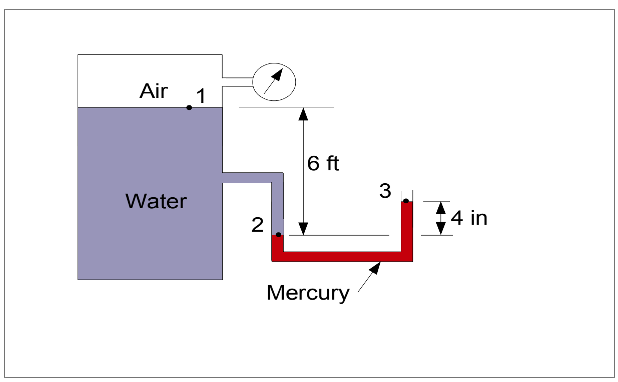


Therefore: $p_3 = p_2 - \gamma_{Hg}(z_3 - z_2)$, but $p_2 = p_1$ and $p_3 = p_4$ (because $\gamma_{air} \ll \gamma_{Hg}$), hence:

$$p_4 = p_1 - \gamma_{Hg}(z_3 - z_2)$$
$$= 15\frac{\text{lbf}}{\text{in}^2} - 13.6 \times 0.0361\frac{\text{lbf}}{\text{in}^3} \times 1.5\,\text{ft}\left|\frac{12\,\text{in}}{1\,\text{ft}}\right| \times \sin(30^\circ) = 10.58\,\text{psi}$$

01-14*. The correct answer is (C)

Define points 1 through 3 as on the figure.

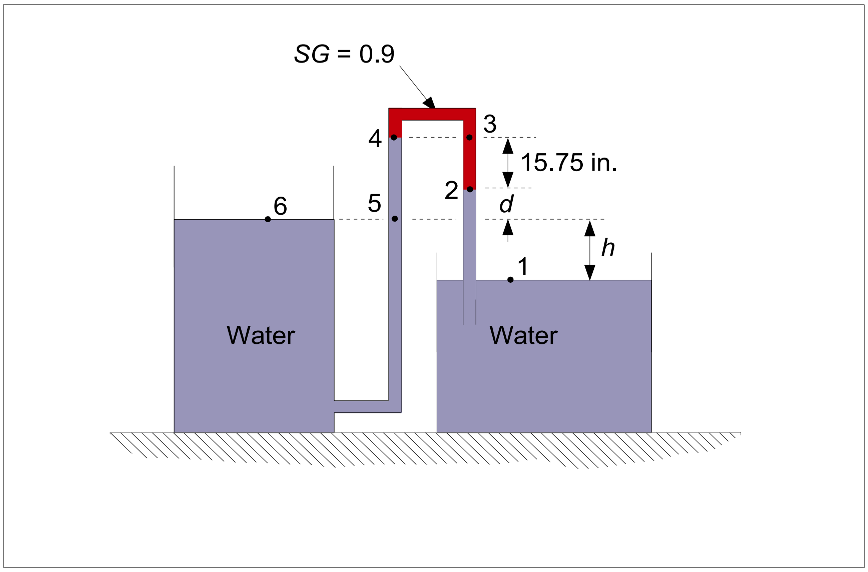


Therefore, $p_2 = p_3 + \gamma_{Hg}(z_3 - z_2)$ but, since $p_3 = 0\,\text{psig}$, we have: $p_2 = \gamma_{Hg}(z_3 - z_2)$.

Similarly, we have: $p_2 = p_1 + \gamma_{H_2O}(z_1 - z_2)$. Now combine these two equations to eliminate p_2:

$$p_1 = \gamma_{Hg}(z_3 - z_2) - \gamma_{H_2O}(z_1 - z_2)$$
$$= 13.6 \times 0.0361\frac{\text{lbf}}{\text{in}^3} \times 4\,\text{in} - 0.0361\frac{\text{lbf}}{\text{in}^3} \times 6\,\text{ft} \times \left|\frac{12\,\text{in}}{1\,\text{ft}}\right| = -0.635\,\text{psi}$$

01-15*. The correct answer is (A). Define points 1 through 6 as in the figure, as well as the distance d



$$p_2 = p_1 - \gamma\left(z_2 - z_1\right) \quad \Rightarrow \quad p_2 = -\gamma\left(d + h\right)$$

Also,

$$p_3 = p_2 - SG\ \gamma\left(z_3 - z_2\right) \quad \Rightarrow \quad p_3 = p_2 - 0.9\ \gamma \times 15.75\ \text{in}$$

Now combine these two expressions (to eliminate p_2):

$$p_3 = -\gamma\left(d + h\right) - 0.9\ \gamma \times 15.75\ \text{in}$$

Since $p_3 = p_4$ then:

$$p_4 = -\gamma\left(d + h\right) - 0.9\ \gamma \times 15.75\ \text{in}$$

Also, for the water column attached to the left tank:

$$p_4 = p_5 - \gamma\left(z_4 - z_5\right) \quad \Rightarrow \quad p_4 = p_5 - \gamma\left(d + 15.75\ \text{in}\right)$$

So, combining these last two expressions (to eliminate p_4):

$$-\gamma\left(d + h\right) - 0.9\ \gamma \times 15.75\ \text{in} \ = \ p_5 - \gamma\left(d + 15.75\ \text{in}\right)$$

Since $p_5 = p_6 = 0$ then:

$$\begin{aligned}
&-\gamma\left(d + h\right) - 0.9\ \gamma \times 15.75\ \text{in} = -\gamma\left(d + 15.75\ \text{in}\right) \\
&-\gamma\, h - 0.9\ \gamma \times 15.75\ \text{in} = -\gamma \times 15.75\ \text{in} \\
&\Rightarrow h = 0.1 \times 15.75\ \text{in} = 1.575\ \text{in} \times \left|\frac{1\ \text{ft}}{12\ \text{in}}\right| = 0.13\ \text{ft}
\end{aligned}$$

01-16*. The correct answer is (C)

Look at Figure 1-4 for a schematic illustration of the situation described in this problem. From Equation 1-5 we have:

$$\tau = \mu \left|\frac{du}{dy}\right| = \mu \frac{U}{b}$$

Since the shear stress $\tau = F/A$ then we can combine with the expression above to obtain:

$$F = A\,\mu\,\frac{U}{b} = 1.4\,\text{ft}^2 \times 6.6\times10^{-4}\,\text{reyn} \times \left|\frac{1\,\dfrac{\text{lbf s}}{\text{in}^2}}{1\,\text{reyn}}\right| \times \frac{3\,\dfrac{\text{ft}}{\text{s}}}{0.1\,\text{in}} = 0.02772\,\text{lbf}\,\frac{\text{ft}^3}{\text{in}^3} \times \left|\frac{12\,\text{in}}{1\,\text{ft}}\right|^3 = 47.9\,\text{ft}$$

01-17*. The correct answer is (D).

The velocity of the fluid varies (linearly, we are assuming) from 10 ft/s to zero across a gap in the radial direction of 0.012-inches. Thus, the magnitude of the velocity gradient is

$$\left|\frac{du}{dr}\right| = \frac{10\,\dfrac{\text{ft}}{\text{s}} \times \left|\dfrac{12\,\text{in}}{1\,\text{ft}}\right|}{0.012\,\text{in}} = 10{,}000\,\text{s}^{-1}$$

Hence, the shear stress on the shaft surface (or, really anywhere within the fluid) is

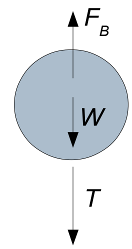
$$\begin{aligned}
\tau &= \mu \left|\frac{du}{dr}\right| \\
&= \nu\,\rho \left|\frac{du}{dr}\right| \\
&= \nu\,SG\,\rho_{H_2O} \left|\frac{du}{dr}\right| \\
&= 0.0086\,\frac{\text{ft}^2}{\text{s}} \times 0.91 \times 62.4\,\frac{\text{lbm}}{\text{ft}^3} \times 10{,}000\,\frac{1}{\text{s}} \;=\; 4883.4\,\frac{\text{lbm}}{\text{ft s}^2} \times \left|\frac{1\,\text{lbf}}{\left(32.174\,\dfrac{\text{lbm}\cdot\text{ft}}{\text{s}^2}\right)}\right| = 151.8\,\frac{\text{lbf}}{\text{ft}^2}
\end{aligned}$$

This shear stress results from the force applied in the direction of travel over the fluid-shaft contact area:

$$\begin{aligned}
F &= \tau\,A \\
&= \tau\,\pi\,D\,L \\
&= 151.8\,\frac{\text{lbf}}{\text{ft}^2} \times \pi \times \left(\frac{1}{12}\right)\text{ft} \times 1.65\,\text{ft} = 65.6\,\text{lbf}
\end{aligned}$$

01-18*. The correct answer is (B)

First draw a free-body diagram of the buoy as shown, where F_B is the buoyant force acting on the buoy, W is the weight of the buoy, and T is the tension in the cable.



For equilibrium it follows that $F_B = W + T \Rightarrow T = F_B - W$. From Equation 1-8:

$$F_B = \gamma \not{V} = 1.025 \times 62.4 \frac{\text{lbf}}{\text{ft}^3} \times \left[(\pi/6)(5\,\text{ft})^3\right] = 4{,}186.2\,\text{lbf}$$

The tension can now be calculated as $T = F_B - W = 4{,}186.2 - 1{,}900 = 2{,}286.2\,\text{lbf}$.

01-19*. The correct answer is (C).

Since the ice block is stationary, the forces acting on it are in balance. Therefore, the buoyancy force which acts vertically up, must equal the weight which acts vertically down:

$$F_B = W$$

where F_B is the weight of the displaced volume of seawater and W is the weight of the ice block. If the side of the cube is L then $W = \gamma_{\text{ice}} L^3$. Also, the volume of displaced seawater is $L^3 - z L^2$ where $z = 10\,\text{in}$ so $F_B = \gamma_{\text{sea}}(L^3 - z L^2)$. Plug all this in the equation of equilibrium:

$$\gamma_{\text{sea}}(L^3 - z L^2) = \gamma_{\text{ice}} L^3$$

Divide through by L^3 ,

$$\gamma_{\text{sea}}\left(1 - \frac{z}{L}\right) = \gamma_{\text{ice}}$$

and solve for L and plug in the numbers:

$$L = \frac{z}{1 - (\gamma_{\text{ice}}/\gamma_{\text{sea}})} = \frac{10\,\text{in}}{1 - (0.92/1.025)} = 97.6\,\text{in}$$

Therefore $z = 97.6 - 10 = 87.6\,\text{in}$

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02: Classification of Flows, Terminology & Bernoulli Equation

If in a given flow field the velocity vector only depends on one spatial coordinate (e.g., x, y, or z) we say it is a **one-dimensional flow**. As examples, consider the flow pattern that occurs in long, straight circular pipes or between parallel plates as shown in Figure 2-1. For the pipe flow, the velocity only depends on the radial coordinate, r, and in the parallel plates case, the velocity only depends on the vertical coordinate y.

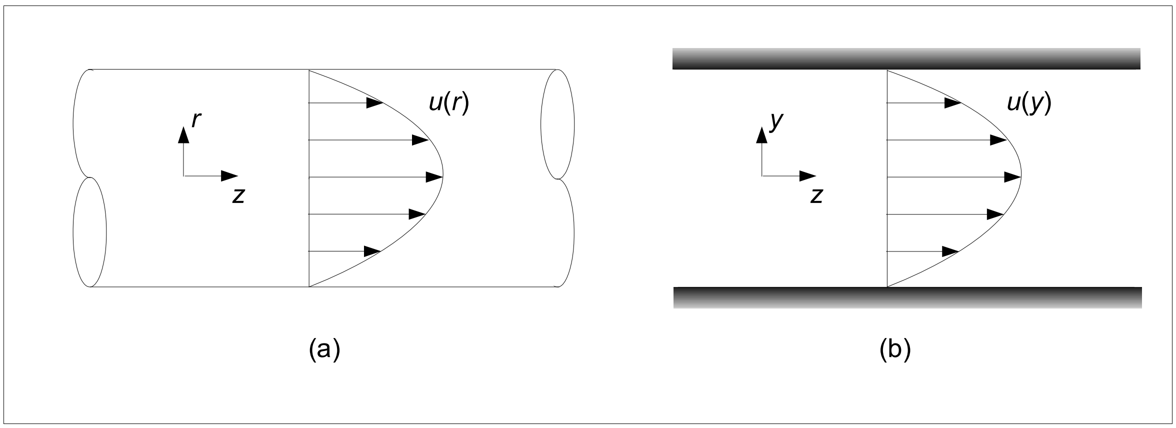


Figure 2-1: One-dimensional flow: (a) flow in a pipe, (b) flow between parallel plates

In the cases above, even if the flow is unsteady (i.e. time-dependent, as in a startup or shutdown) the flow would still be one-dimensional. The velocity profiles shown in Figure 2-1 can also be referred to as **fully developed** flows, which means that the velocity profiles do not change in the direction of flow (i.e., the z direction in the figure).

Many times in engineering applications considerable simplicity is achieved by invoking the assumption of **uniform flow**, which implies that the velocity is <u>constant</u> over the cross-sectional area of the pipe or conduit. The average velocity may change from one section to the other, but the flow conditions only depend on the space variable in the direction of flow – as in Figure 2-2.

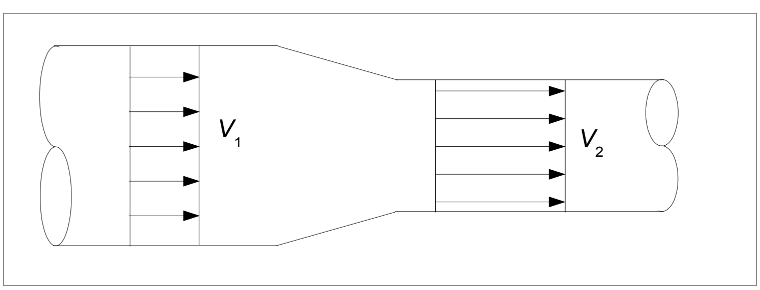


Figure 2-2: Uniform velocity profiles

In a broad sense, a fluid flow may be classified as inviscid or viscous. An **inviscid flow** is one in which the effects of the fluid's viscosity does not significantly influence the flow field, so these effects are neglected. The primary class of flows that can be accurately analyzed as inviscid are **external flows**, that is, flows that exist exterior to a solid body such as flow around an airfoil or a vehicle. Any viscous effects that may exist are all within a very thin region adjacent to the surface of the solid body known as a **boundary layer**. In many practical applications, the boundary layers are so thin that they can be ignored when studying the gross, macroscopic features of a flow. **Viscous flows** include **internal flows**, such as flows in pipes, conduits, and open channels. Viscous effects are responsible for the "losses" encountered in these flows which we will examine in greater detail in further sections.

Viscous flows can be further classified as laminar or turbulent. In **laminar flow**, there is no significant mixing of fluid particles with their neighboring particles. Imagine a set of "sheets" sliding past each other. We recommend googling "laminar flow visualization" videos for some impressive demonstrations of this feature. In many of these videos, you'll see a dye being injected in the flow. The dye does not mix, retaining its "identity" for a relatively long time, over relatively long distances. The flow may be time-dependent, or it may be steady. In a **turbulent flow**, quantities such as velocity and pressure show random fluctuations with time and with the space coordinates. For turbulent conditions, a flow is deemed "steady" if the time-averaged physical quantities (e.g., pressure, velocity, temperature, etc) do not change in time. If you were to measure the velocity magnitude of a steady, turbulent flow in a given location, you would notice an irregular pattern made by all the instantaneous values, but they would all be "around" an average value that doesn't change with time. A dye injected into a turbulent flow would quickly mix by the action of randomly moving particles.

The interaction of three key parameters defines if a fluid flow is laminar or turbulent. These three parameters are the length scale, the fluid velocity scale, and the fluid's viscosity. The three parameters are combined into a single dimensionless parameter (that can serve as a tool to make predictions about the flow being laminar or turbulent) known as the **Reynolds number**:

$$\mathrm{Re} = \frac{V\,L}{\nu} \tag{2-1}$$

where V and L are a characteristic velocity and length, respectively, and ν is the fluid's kinematic viscosity. If the Reynolds number is relatively low, the flow will be laminar; if it is large, the flow will be turbulent. This phenomenon is quantified with the use of a **critical Reynolds number**, Re_{crit} , so that the flow is laminar if $\mathrm{Re} < \mathrm{Re}_{crit}$. The magnitude of Re_{crit} depends on the flow configuration, for example for flow inside a rough walled pipe in most engineering applications, a value $\mathrm{Re}_{crit} = 2{,}000$ is typically used. A simple example of a transition from laminar to turbulent may be observed on the smoke rising from a cigarette or a smoke stack. For a certain distance, the smoke rises in a smooth (laminar) manner but then abruptly, the smoke mixes up and becomes turbulent and the narrow smoke column widens and diffuses.

Finally, another flow classification distinguishes flows that are compressible from those that are incompressible. An **incompressible flow** exists if the density of each fluid particle remains relatively constant as it moves through the flow field. For most practical purposes, liquid flows are incompressible. Also, under certain conditions involving minor pressure changes and low velocities, gas flows may be considered incompressible. If the gas experiences relatively large density changes which influence the flow, then it is **compressible** flow. For compressible flow, there is a great distinction between flows involving velocities less than that of sound (**subsonic** flows) and flow involving velocities greater than that of sound (**supersonic** flows).[6] The **Mach number** is a measure of the flow characteristic velocity relative to the local speed of sound:

$$M = \frac{V}{c} \tag{2-2}$$

where V is the gas speed and c is the local speed of sound. Thus, when $M < 1$ the flow is subsonic, and when $M > 1$ the flow is supersonic. In gases, the speed of sound is given by $c = \sqrt{k\,RT}$ where T is the absolute temperature, k is a property known as the specific heat ratio, and R is the particular gas constant.[7] For air $k = 1.4$, and the gas constant is:

$$R_{air} = \begin{Bmatrix} 53.34\ \mathrm{ft \cdot lbf/lbm \cdot {}^\circ R} \\ 0.3704\ \mathrm{psia \cdot ft^3/lbm \cdot {}^\circ R} \\ 640.08\ \mathrm{psia \cdot in^3/lbm \cdot {}^\circ R} \\ 1{,}716.2\ \mathrm{ft^2/s^2 \cdot {}^\circ R} \\ 0.0686\ \mathrm{Btu/lbm \cdot {}^\circ R} \\ 0.287\ \mathrm{kJ/kg \cdot K} \end{Bmatrix}$$

When $M < 0.3$ it can be shown that density variations are not important so $\mathrm{M} < 0.3$ (which corresponds roughly to a velocity under 300 ft/s or 100 m/s for standard air) is typically used as a criterion to decide

6 Sonic speed in air at 70°F (21.1°C) is about 1,128 ft/s (343.9 m/s)

7 For a thorough review of the ideal gas law please consult our Thermodynamics and Energy Balances ebook.

to model flow as incompressible without any significant loss of accuracy.

The well-known **Bernoulli equation**[8] adopts the following form when applied between two points on the same streamline:

$$\frac{V_1^2}{2}+\frac{p_1}{\rho}+g\,z_1=\frac{V_2^2}{2}+\frac{p_2}{\rho}+g\,z_2 \tag{2-3}$$

If we divide the equation above by g, it becomes:

$$\frac{V_1^2}{2\mathrm{g}}+\frac{p_1}{\gamma}+z_1=\frac{V_2^2}{2\mathrm{g}}+\frac{p_2}{\gamma}+z_2 \tag{2-4}$$

The term $V^2/2\,g$ is known as the **velocity head** and it represents the vertical distance needed for the fluid to fall freely (neglecting friction) if it is to reach velocity V from rest. The term $p/\,\gamma$ is known as the **pressure head**, and it represents the height of a column of the fluid that is needed to produce the pressure p . The elevation term z is related to the potential energy of the fluid and it is known as the **elevation head**. The sum $(p/\,\gamma)+z$ is called **piezometric head**, and the sum of all three terms is the **total head**. The pressure p is sometimes called **static pressure** and the group $p+(1/2)\rho\,V^2$ is called the **stagnation pressure**. The group $p+(1/2)\rho\,V^2+\gamma\,z$ is called **total pressure**. That is, total pressure is the sum of static, dynamic, and hydrostatic pressures. Consider the arrangement shown in Figure 2-3. Here we see a pressure gage (or **piezometer**) mounted on the wall of a pipe or conduit, measuring the static pressure at location 1. The probe on the right is a pitot-tube which is a thin, dead-end conduit inserted in the flow. The fluid inside the tube (point 2) is stagnant.

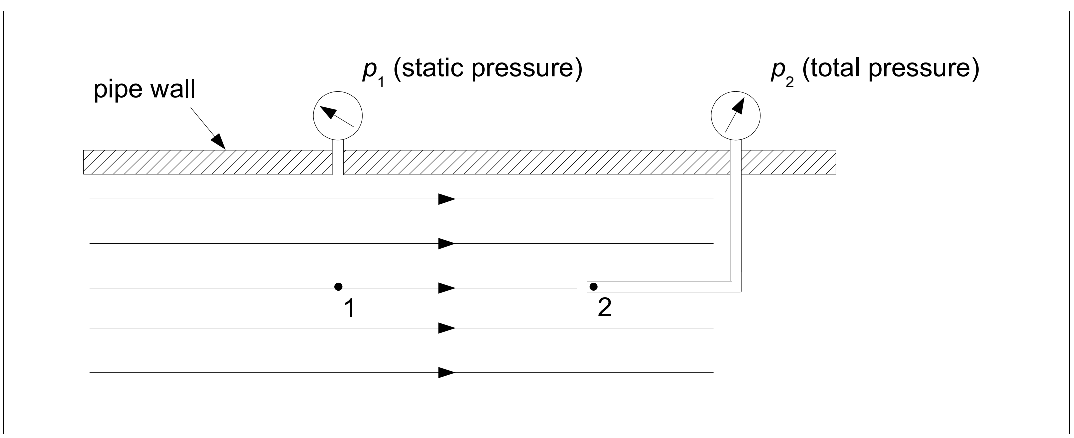


Figure 2-3: Pressure probes: piezometer (for static pressure) and pitot tube (for total pressure).

8 This equation is obtained by applying F = ma along a streamline for steady, inviscid, incompressible flow.

If we apply Equation 2-3 from point 1 to point 2 we see that the pressure read by the pitot probe is the stagnation pressure. The difference between the readings of the two probes can be used to determine the velocity at point 1. Another device, known as a **pitot-static probe** combines both devices into one instrument. The pitot-static probe provides the value of $p_2 - p_1$. As an exercise, apply the Bernoulli equation between points 1 and 2 in Figure 2-3 to show that the velocity at point 1 is given by:

$$V_1 = \sqrt{\frac{2}{\rho}(p_2 - p_1)}$$

The Bernoulli equation can be used on many steady flow situations of engineering interest, as long as viscous and compressibility effects are not important. Examples of external flows where the Bernoulli equation is typically applied include determining the height of a jet of water, the force applied on windows due to wind, or the surface pressure on a low velocity airfoil.

Examples of internal flows where the Bernoulli equation is typically applied involve internal flows over short distances such as flow through contractions and flow from a plenum, nozzles and in pipes of variable diameter for which the fluid velocity changes because the flow area is different from one section to another. **If you search the PEMH for "Bernoulli" you'll find Equation 2-4 but there it includes an additional term for friction loss. We discuss this additional term in subsequent sections.**

In this section, we focus on situations as described in the previous two paragraphs were friction effects are negligible. Often it is necessary to use the concept of conservation of mass (the continuity equation) along with the Bernoulli equation. For a control volume in steady state, **conservation of mass** requires that the rate at which the mass of fluid flows into the volume must equal the rate at which it flows out of the volume (otherwise, mass would not be conserved). The mass flow rate $\dot{m}$ (lbm/h, lbm/min, kg/h, etc) is given by $\dot{m} = \rho Q$ where Q is the volume flow rate (gal/min, ft^3/min, m^3/s, etc). If at a given section the cross-sectional area is A, and the flow occurs across this area (normal to the area) with an average velocity V the volume flow rate can be shown to be given by $Q = AV$. If we label the inlet to the control volume as "1" and the outlet as "2", then conservation of mass requires: $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$. If the density remains constant, then $\rho_1 = \rho_2$ and the above becomes the continuity equation for incompressible flow:

$$A_1 V_1 = A_2 V_2 \qquad (2\text{-}5)$$

Refer to Figure 2-2. A reduction in the cross sectional area brings about an increase in the velocity.

PROBLEMS:

02-01. A 4-in. ID pipe carries 300 gpm of water at a pressure of 30 psig. Determine – in feet of water – (a) the pressure head, (b) the velocity head, and (c) the total head with reference to a datum plane 20 ft below the pipe.

02-02. Water flows at a rate of 300 gpm in a 4-inch, schedule-40, seamless steel pipe (ID=4.026 inches). The pipe is reduced down to a 3-inch, schedule-40, seamless steel pipe (ID=3.068 inches). What is the percentage change in velocity head after the reduction? What is the percentage change in pressure head after the reduction?

02-03*. A pitot tube measures the velocity of a small aircraft flying at an altitude of 3,000 ft., where atmospheric pressure and temperature are 13.2 psia and 48°F respectively, and the air density is 0.00218 slugs/ft^3. The pitot tube reading is 0.7 psi. Under these conditions, the speed of the aircraft (miles per hour) is most nearly:

(A) 192

(B) 282

(C) 880

(D) 921

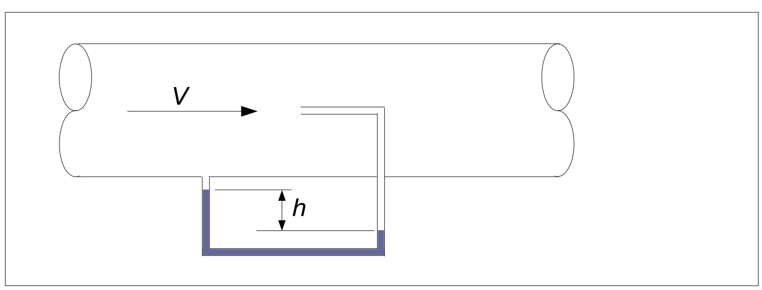
02-04*. The velocity (feet per second) of the water in the pipe, when *h*=4 inches of Hg is most nearly:

(A) 0.25

(B) 1.42

(C) 3.0

(D) 17.1



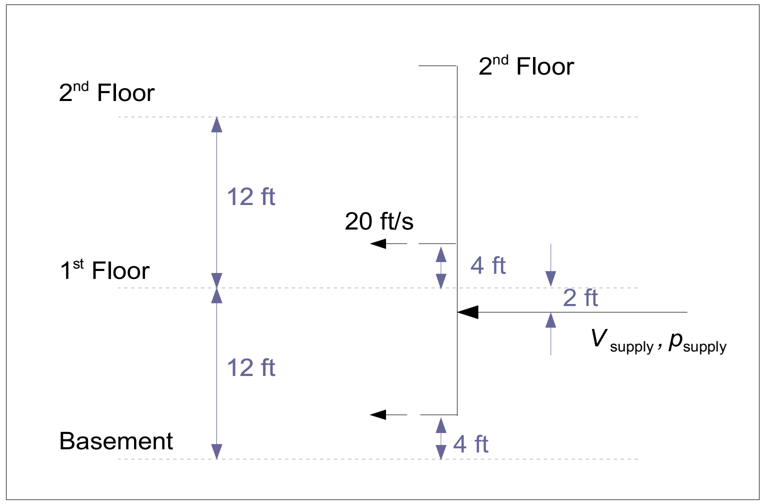
02-05*. Water is supplied to a house at pressure p_{supply} while flowing at a velocity V_{supply}. The maximum velocity of water coming out of a faucet in the first floor is 20 ft/s. Neglect any friction losses (e.g., assume inviscid flow). Under these conditions the maximum water velocity that would be expected of the water (ft/s) coming out of the basement faucet is most nearly:

(A) 23

(B) 34

(C) 45

(D) 50



02-06*. A fire hose nozzle has a diameter of 1.125 inches. According the local fire code, the nozzle must be capable of delivering at least 250 gpm when attached to a 3-in.-diameter hose. Under these conditions, the pressure (psig) that must be maintained just upstream of the nozzle is most nearly:

(A) 35

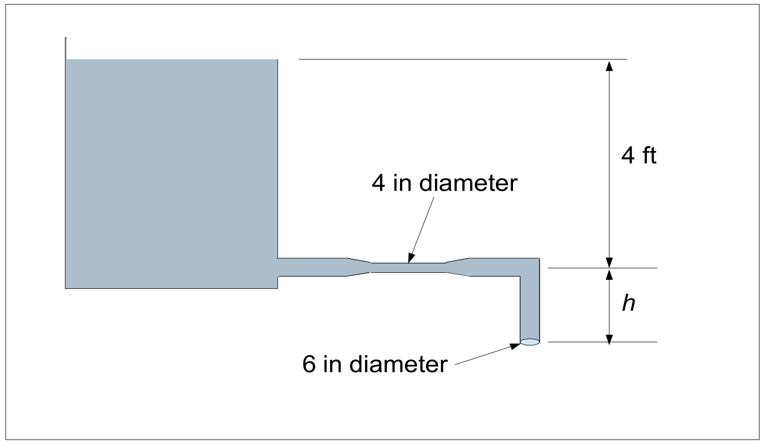
(B) 43

(C) 52

(D) 6,189

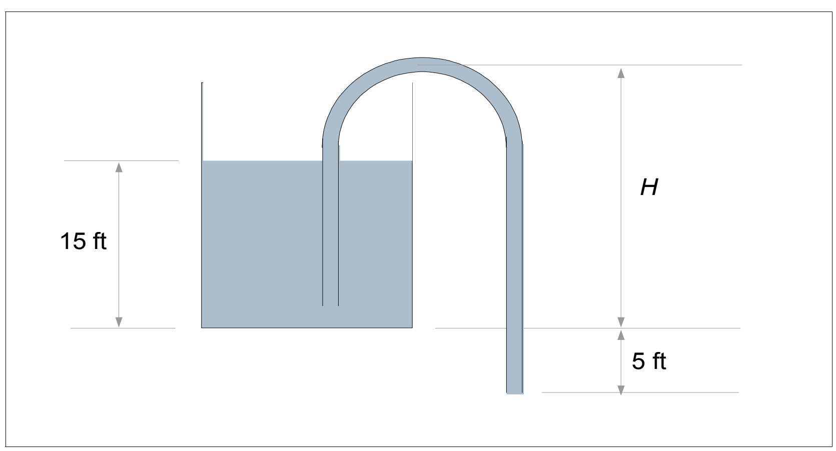
02-07*. Water flows steadily from a large tank, with negligible friction effects. The 4-in diameter section of the thin wall tubing will collapse if the pressure there drops to 10 psi below atmospheric pressure. Under these conditions, the largest allowable value of *h* (inches) is most nearly:

(A) 1.4
(B) 6
(C) 10
(D) 17



02-08*. Water at 60°F is being syphoned with a constant diameter hose from a large tank open to atmosphere, where the local atmospheric pressure is 14.7 psia. More information is provided in the picture. Neglecting all friction effects, the maximum height of the hill, H (ft) over which the water can be siphoned without cavitation (localized formation of water vapor bubbles in the hose due to evaporation) occurring is most nearly:

(A) 21
(B) 28
(C) 36
(D) 44



Solutions:

02-01.

(a) The pressure head is 30 psig expressed in feet of water. This is done by using Equation 1-4:

$$h_p = \frac{p}{\gamma_{\mathrm{H_2O}}} = \frac{30\,\dfrac{\mathrm{lbf}}{\mathrm{in}^2}}{0.0361\,\dfrac{\mathrm{lbf}}{\mathrm{in}^3}} = 831\,\mathrm{in} \times \left|\frac{1\,\mathrm{ft}}{12\,\mathrm{in}}\right| = 69.3\,\mathrm{ft}$$

(b) First, determine the average velocity from the given flow rate:

$$V = \frac{Q}{A_{\mathrm{pipe}}} = \frac{Q}{\frac{\pi}{4}D^2} = \frac{300\,\mathrm{gpm} \times \left|\dfrac{1\,\mathrm{ft}^3/\,\mathrm{s}}{448.831\,\mathrm{gpm}}\right|}{\frac{\pi}{4}(4\,\mathrm{in})^2 \times \left|\dfrac{1\,\mathrm{ft}^2}{144\,\mathrm{in}^2}\right|} = 7.659\,\frac{\mathrm{ft}}{\mathrm{s}}$$

Now, the velocity head is found from its definition:

$$h_V = \frac{V^2}{2g} = \frac{\left(7.659\,\dfrac{\mathrm{ft}}{\mathrm{s}}\right)^2}{2 \times 32.2\,\dfrac{\mathrm{ft}}{\mathrm{s}^2}} = 0.91\,\mathrm{ft}$$

(c) The total head is the sum of the velocity, pressure and elevation heads:

$$h_T = h_p + h_v + z = 69.3\,\mathrm{ft} + 0.91\,\mathrm{ft} + 20\,\mathrm{ft} = 90.21\,\mathrm{ft}$$

02-02. (a) First determine the average velocity in each section:

$$V_1 = \frac{Q}{A_{\mathrm{pipe}}} = \frac{300\,\mathrm{gpm} \times \left|\dfrac{1\,\mathrm{ft}^3/\,\mathrm{s}}{448.831\,\mathrm{gpm}}\right|}{\frac{\pi}{4}(4.026\,\mathrm{in})^2 \times \left|\dfrac{1\,\mathrm{ft}^2}{144\,\mathrm{in}^2}\right|} = 7.56\,\frac{\mathrm{ft}}{\mathrm{s}}$$

$$V_2 = \frac{Q}{A_{\mathrm{pipe}}} = \frac{300\,\mathrm{gpm} \times \left|\dfrac{1\,\mathrm{ft}^3/\,\mathrm{s}}{448.831\,\mathrm{gpm}}\right|}{\frac{\pi}{4}(3.068\,\mathrm{in})^2 \times \left|\dfrac{1\,\mathrm{ft}^2}{144\,\mathrm{in}^2}\right|} = 13.02\,\frac{\mathrm{ft}}{\mathrm{s}}$$

Now calculate the respective velocity heads:

$$h_{V_1} = \frac{V_1^2}{2g} = \frac{\left(7.56\,\dfrac{\mathrm{ft}}{\mathrm{s}}\right)^2}{2 \times 32.2\,\dfrac{\mathrm{ft}}{\mathrm{s}^2}} = 0.89\,\mathrm{ft}$$

$$h_{V_2} = \frac{V_2^2}{2g} = \frac{\left(13.02\,\dfrac{\mathrm{ft}}{\mathrm{s}}\right)^2}{2 \times 32.2\,\dfrac{\mathrm{ft}}{\mathrm{s}^2}} = 2.63\,\mathrm{ft}$$

So the change in velocity head is an increase of $2.63\,\mathrm{ft} - 0.89\,\mathrm{ft} = 1.74\,\mathrm{ft}$.

(b) To determine the change in pressure head use the Bernoulli equation (which can be interpreted as the statement that total head remains constant) assuming negligible change in elevation:

$$\frac{V_1^2}{2g}+\frac{p_1}{\gamma}=\frac{V_2^2}{2g}+\frac{p_2}{\gamma}$$
$$\Rightarrow\frac{p_2}{\gamma}-\frac{p_1}{\gamma}=-\left(\frac{V_2^2}{2g}-\frac{V_1^2}{2g}\right)$$

That is, the change in pressure head is equal but opposite the change in velocity head. Therefore, the change in pressure head is a *decrease* of 1.74 ft.

02-03*. The correct answer is (A)

Place a frame of reference on the airplane. This way, the pitot tube is stationary and the far-field velocity magnitude of the air is the aircraft speed. The reading from the pitot tube is the stagnation, or total, pressure. See Equation 2-3.

$$p_T=p+\rho\frac{V^2}{2}\;\;\Rightarrow\;\;V=\sqrt{\frac{2(p_T-p)}{\rho}}$$

If we work with gage pressures, we set $p=0$. Therefore:

$$V=\sqrt{\frac{p_T-p}{\rho}}=\sqrt{\frac{2\times 0.6\,\dfrac{\text{lbf}}{\text{in}^2}\times\left|\dfrac{12\text{ in}}{1\text{ ft}}\right|^2}{0.00218\,\dfrac{\text{slugs}}{\text{ft}^3}}\times\left|\frac{1\text{ ft/s}^2}{1\text{ lbf/slugs}}\right|}=281.5\,\frac{\text{ft}}{\text{s}}$$
$$=281.5\,\frac{\text{ft}}{\text{s}}\times\left|\frac{3{,}600\text{ s}}{1\text{ h}}\right|\times\left|\frac{1\text{ mile}}{5{,}280\text{ ft}}\right|=192\text{ mph}$$

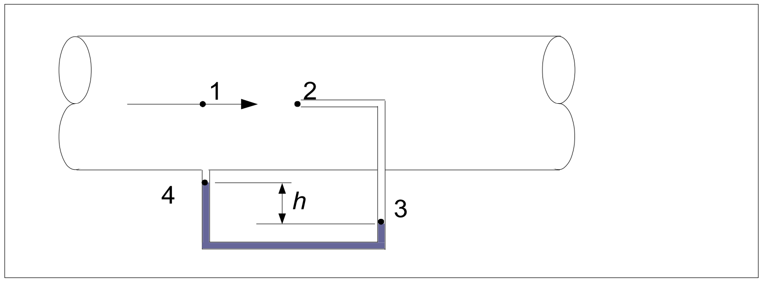
At this point, it is a good idea to check the magnitude of the Mach number, to ensure that compressibility effects are negligible and the results obtained from the Bernoulli equation are accurate. The speed of sound is:

$$c=\sqrt{k\,R_{\text{air}}\,T}=\sqrt{1.4\times1{,}716.2\,\frac{\text{ft}^2}{\text{s}^2\,°\text{R}}\times(48+459.67)\,°\text{R}}=1{,}105.6\,\frac{\text{ft}}{\text{s}}$$

Therefore the Mach number is $\text{M}=V/c=281.5/1{,}105.6=0.255$. Since $\text{M}<0.3$ compressibility effects are not important and the Bernoulli equation can be applied. However, note that it is very close, so if the velocity were to increase, the equations of compressible flow should be employed.

02-04*. The correct answer is (D).

Define points 1 through 4 as in the figure:



Apply the Bernoulli equation from point 1 to point 2 (which is a stagnation point) and solve for the velocity at 1:

$$V_1 = \sqrt{\frac{2(p_2 - p_1)}{\rho}}$$

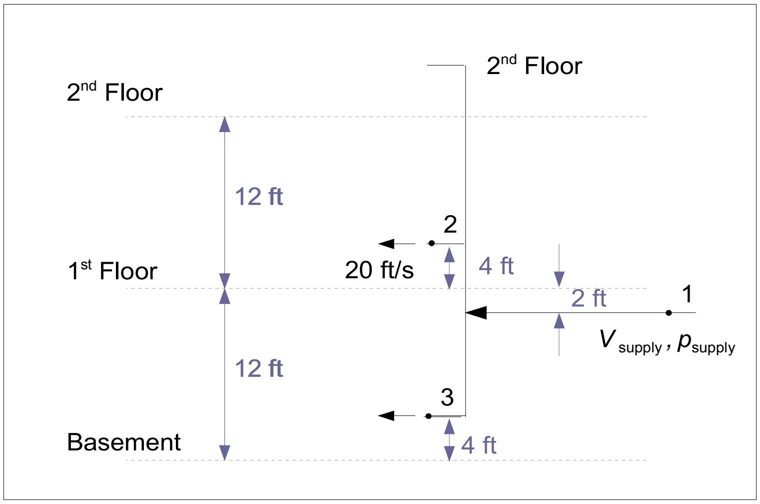
If we neglect the weight of the water column of height h above point 3, then:

$$(p_2 - p_1) = (p_3 - p_4)$$

where $(p_3 - p_4) = \gamma_{\mathrm{Hg}} h = 13.6 \times 0.0361 \frac{\mathrm{lbf}}{\mathrm{in}^3} \times 4\,\mathrm{in} = 1.964 \frac{\mathrm{lbf}}{\mathrm{in}^2}$. Therefore, the velocity is:

$$V_1 = \sqrt{\frac{2(p_2 - p_1)}{\rho}} = \sqrt{\frac{2 \times 1.964 \frac{\mathrm{lbf}}{\mathrm{in}^2} \times \left| \frac{144\,\mathrm{in}^2}{1\,\mathrm{ft}^2} \right|}{62.4 \frac{\mathrm{lbm}}{\mathrm{ft}^3} \times \left| \frac{1\,\mathrm{lbf}}{\left(32.174 \frac{\mathrm{lbm \cdot ft}}{\mathrm{s}^2}\right)} \right|}} = 17.1 \frac{\mathrm{ft}}{\mathrm{s}}$$

02-05.* Define points 1, 2, and 3 as in the figure.



Apply the Bernoulli equation from 1 to 2 (note we will set $p_2 = 0$)

$$\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

$$\Rightarrow \frac{V_{\text{supply}}^2}{2g} + \frac{p_{\text{supply}}}{\gamma} = \frac{V_2^2}{2g} + (z_2 - z_1)$$

Similarly, when we apply Bernoulli from 1 to 3 (with $p_3 = 0$)

$$\frac{V_{\text{supply}}^2}{2g} + \frac{p_{\text{supply}}}{\gamma} = \frac{V_3^2}{2g} - (z_1 - z_3)$$

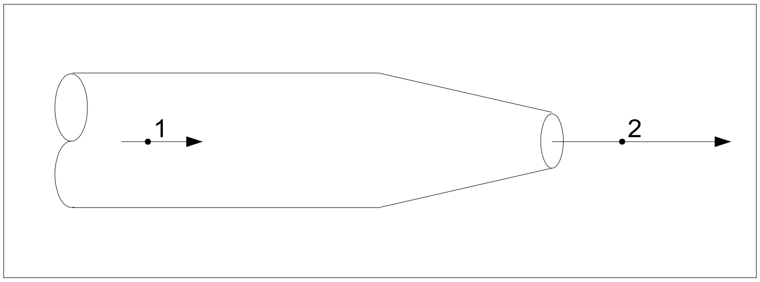
We can equate the last two expressions;

$$\frac{V_3^2}{2g} - (z_1 - z_3) = \frac{V_2^2}{2g} + (z_2 - z_1)$$

$$\Rightarrow \frac{V_3^2}{2g} = \frac{V_2^2}{2g} + (z_2 - z_3)$$

$$\Rightarrow V_3 = \sqrt{V_2^2 + 2g(z_2 - z_3)} = \sqrt{\left(20\,\frac{\text{ft}}{\text{s}}\right)^2 + 2 \times 32.2\,\frac{\text{ft}}{\text{s}^2} \times 12\,\text{ft}} = 34.25\,\frac{\text{ft}}{\text{s}}$$

02-06.* Define point 1 as inside the hose, upstream of the nozzle and point 2 as in the nozzle discharge, at atmospheric pressure:



The pressure at point 1 can be obtained from the Bernoulli equation applied from 1 to 2 (using $p_2=0$, and $z_2=z_1$)

$$\frac{V_1^2}{2\mathrm{g}}+\frac{p_1}{\gamma}=\frac{V_2^2}{2\mathrm{g}}+\frac{p_2}{\gamma} \quad \Rightarrow \quad p_1=\rho\,\frac{V_2^2-V_1^2}{2}$$

So, we see we need the velocities at 1 and 2. The velocity upstream of the nozzle (section 1) is:

$$V_1=\frac{Q}{A_{\text{hose}}}=\frac{250\ \text{gpm}\times\left|\dfrac{1\,\text{ft}^3/\ \text{s}}{448.83\,\text{gpm}}\right|}{\dfrac{\pi}{4}3\,\text{in}^2\times\left|\dfrac{1\,\text{ft}^2}{144\,\text{in}^2}\right|}=11.34\,\frac{\text{ft}}{\text{s}}$$

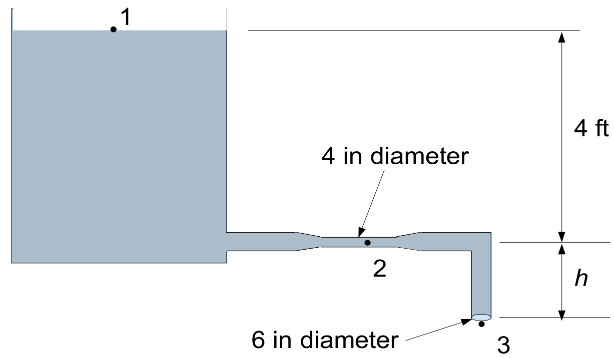
Now we can use the continuity equation to get the nozzle discharge velocity:

$$A_{\text{nozzle}}V_2=A_{\text{hose}}V_1$$
$$\Rightarrow V_2=\left(D_1/D_2\right)^2V_1=\left(3/1.125\right)^2 13.6\,\text{ft/s}=80.69\,\frac{\text{ft}}{\text{s}}$$

We can plug these in the expression for p_1

$$p_1=\rho\,\frac{V_2^2-V_1^2}{2}=62.4\,\frac{\text{lbm}}{\text{ft}^3}\times\left|\frac{1\,\text{lbf}}{\left(32.174\,\dfrac{\text{lbm}\cdot\text{ft}}{\text{s}^2}\right)}\right|\times\frac{\left(80.69^2-11.34^2\right)\dfrac{\text{ft}^2}{\text{s}^2}}{2}\times\left|\frac{1\,\text{ft}^2}{144\,\text{in}^2}\right|=42.9\,\frac{\text{lbf}}{\text{in}^2}$$

02-07.* The correct answer is (D). Define points 1, 2, and 3 as in the figure:



Using gage pressures we know that $p_1 = p_3 = 0\,\text{psig}$ and $p_2 = -10\,\text{psig}$.

Apply the Bernoulli equation between 2 and 3, and solve for $(z_2 - z_3)$:

$$\frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2 = \frac{V_3^2}{2g} + \frac{p_3}{\gamma} + z_3$$
$$\Rightarrow (z_2 - z_3) = h = \frac{V_3^2}{2g} - \frac{V_2^2}{2g} - \frac{p_2}{\gamma}$$

So we need V_2 and V_3 . Apply the Bernoulli equation between 1 and 2, and solving for V_2 :

$$\frac{V_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{V_2^2}{2g} + \frac{p_2}{\gamma} + z_2$$

$$\Rightarrow \quad \frac{V_2^2}{2g} = (z_1 - z_2) - \frac{p_2}{\gamma} \;=\; 4\,\text{ft} - \frac{-10\,\frac{\text{lbf}}{\text{in}^2}}{0.0361\,\frac{\text{lbf}}{\text{in}^3}} \times \left|\frac{1\,\text{ft}}{12\,\text{in}}\right| \;=\; 27.08\,\text{ft} \;\Rightarrow\; V_2 = 41.75\,\frac{\text{ft}}{\text{s}}$$

and now the velocity at 3 is found by invoking the continuity equation between sections 2 and 3:

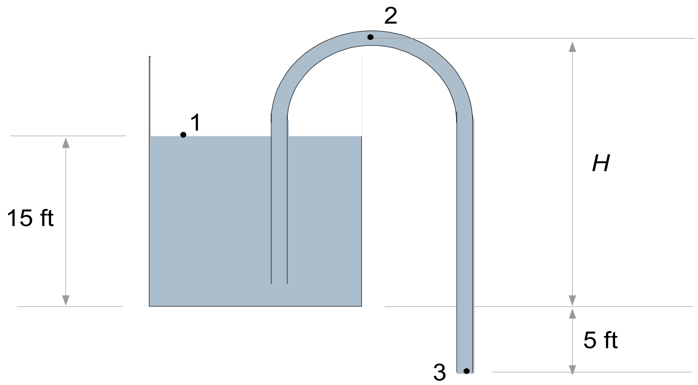
$$A_2 V_2 = A_3 V_3 \;\Rightarrow\; V_3 = \left(\frac{D_2}{D_3}\right)^2 V_2 = \left(\frac{4\,\text{in}}{6\,\text{in}}\right)^2 \times 41.75\,\frac{\text{ft}}{\text{s}} = 18.6\,\frac{\text{ft}}{\text{s}}$$

Now we can insert these values in our expression for h :

$$h = \frac{V_3^2 - V_2^2}{2g} - \frac{p_2}{\gamma}$$
$$= \frac{(18.6^2 - 41.75^2)\,\frac{\text{ft}^2}{\text{s}^2}}{2 \times 32.2\,\frac{\text{ft}}{\text{s}^2}} \times \left|\frac{12\,\text{in}}{1\,\text{ft}}\right| - \frac{\left(-10\,\frac{\text{lbf}}{\text{in}^2}\right)}{0.0361\,\frac{\text{lbf}}{\text{in}^3}} \quad = 16.6\,\text{in}$$

02-08* The correct answer is (B).

Define points 1, 2 and 3 as in the figure:



So $p_1=0$, $p_3=0$, $z_1=15\,\text{ft}$, $z_2=H$; $z_3=-5\,\text{ft}$

The pressure at 2 should be no lower than the saturation pressure corresponding to 60°F, which is 0.2564 psia. We are, however using gage pressures in this problem, thus $p_3=(0.2564-14.7)=-14.44\,\text{psig}$.

Write the Bernoulli equation between 1 and 2:

$$\frac{V_1^2}{2g}+\frac{p_1}{\gamma}+z_1=\frac{V_2^2}{2g}+\frac{p_2}{\gamma}+z_2$$

and solve for z_2 :

$$z_2=z_1-\frac{p_2}{\gamma}-\frac{V_2^2}{2g}$$

We see that we need the flow velocity at 2. However, from continuity we know that $A_2V_2=A_3V_3$, therefore $V_2=V_3$ (because the hose has constant diameter). In order to find V_3 we can write the Bernoulli equation between 1 and 3 to produce the following equation for the hose discharge velocity:

$$V_3=\sqrt{2g(z_1-z_3)}=\sqrt{2\times 32.2\,\text{ft/s}^2\times(15-(-5))\,\text{ft}}=35.9\,\text{ft/s}$$

Now we can return to our expression for z_2 :

$$z_2=z_1-\frac{p_2}{\gamma}-\frac{V_2^2}{2g}$$

Note that if z_2 increases, then p_2 *has* to decrease. Therefore, when we set $p_2=p_{sat}(60\,°\text{F})$ we are determining the highest allowable value of z_2 .

$$H=15\,\text{ft}-\frac{-14.44\,\dfrac{\text{lbf}}{\text{in}^2}}{0.0361\,\dfrac{\text{lbf}}{\text{in}^3}}-\frac{\left(35.9\,\dfrac{\text{ft}}{\text{s}}\right)^2}{2\times 32.2\,\dfrac{\text{ft}}{\text{s}^2}}$$

$$=15\,\text{ft}+400\,\text{in}\times\left|\frac{1\,\text{ft}}{12\,\text{in}}\right|-20\,\text{ft}\quad\Rightarrow\quad H=28.3\,\text{ft}$$

07-04*. The correct answer is (A).

See Figure 7-2 for reference. The mass flow of makeup water is used to match the water lost by evaporation: $\dot{m}_{\text{makeup}} = \dot{m}_{\text{air}}(W_2 - W_1)$

where $W_2 = 0.037\,\text{lbm/lbm}$ and $W_1 = 0.0043\,\text{lbm/lbm}$ are obtained from the psychrometric chart. To obtain the mass flow rate of air, use the energy balance combined witht the water mass balance as in the previous problems:

$$\dot{m}_{\text{air}} = \frac{\dot{m}_w(h_3 - h_5)}{(h_2 - h_1) - (W_2 - W_1)h_5} = \frac{\dot{m}_w c_{p,\text{w}}(T_3 - T_5)}{(h_2 - h_1) - (W_2 - W_1)h_5}$$

The numerator in the equation above is the tower heat load (which is a given parameter in this problem):

$$\dot{m}_{\text{air}} = \frac{\dot{Q}_{\text{CT}}}{(h_2 - h_1) - (W_2 - W_1)h_5}$$

where $h_1 = 21.1\,\text{Btu/lbm}$, $h_2 = 63.2\,\text{Btu/lbm}$, can be obtained from the psychrometric chart and $h_5 \approx h_f(70\,°\text{F}) = 38.1\,\text{Btu/lbm}$ from the steam tables.

$$\dot{m}_{\text{air}} = \frac{\dot{Q}_{\text{CT}}}{(h_2 - h_1) - (W_2 - W_1)h_5} = \frac{600{,}000\,\dfrac{\text{Btu}}{\text{h}}}{(63.2 - 21.1)\dfrac{\text{Btu}}{\text{lbm}} - (0.037 - 0.0043)\dfrac{\text{lbm}}{\text{lbm}} \times 38.1\,\dfrac{\text{Btu}}{\text{lbm}}} = 14{,}686.4\,\frac{\text{lbm}}{\text{h}}$$

Therefore

$$\dot{m}_{\text{makeup}} = 14{,}686.4\,\frac{\text{lbm}}{\text{h}} \times (0.037 - 0.0043)\frac{\text{lbm}}{\text{lbm}} = 480.3\,\frac{\text{lbm}}{\text{h}}$$

So, the volume flow rate is $(458.9\,\text{lbm/h}) \div (8.34\,\text{lbm/gal}) \approx 58\,\text{gal/h}$

07-05*. The correct answer is (D).

The cooling tower load is given by Equation 7-2:

$$\dot{Q}_{\text{CT}} = \dot{m}_{\text{w}} c_{p,\text{w}} (T_{\text{w,in}} - T_{\text{w,out}})$$

which can be combined with the provided definition of tower cooling efficiency, yielding:

$$\begin{aligned}\dot{Q}_{\text{CT}} &= \dot{m}_{\text{w}} c_{p,\text{w}}\, \eta_{\text{CT}} (T_{\text{w,in}} - T_{\text{wet bulb, in}}) \\ &= \left(30\,\frac{\text{gal}}{\text{min}} \times 8.34\,\frac{\text{lbm}}{\text{gal}}\right) \times \left|\frac{60\,\text{min}}{1\,\text{h}}\right| \times 1\,\frac{\text{Btu}}{\text{lbm}\,°\text{F}} \times 0.75 \times (95 - 45.7)\,°\text{F} = 555{,}069\,\frac{\text{Btu}}{\text{h}}\end{aligned}$$

Part IV: HEAT TRANSFER AND HEAT EXCHANGERS

01: Introduction

Heat Transfer occurs whenever a temperature difference exists in a medium or between media. We refer to different types of heat transfer processes as modes. **Conduction** refers to the heat transfer that occurs across a (solid or fluid) medium. In contrast, the term **convection** refers to heat transfer that occurs between a surface and a moving fluid when they are at different temperatures. Finally, all surfaces of finite temperature emit energy in the form of electromagnetic waves, a phenomenon we call **thermal radiation**. Hence, in the absence of a participating medium, there is net heat transfer by radiation between two surfaces at different temperatures.

When a furnace wall feels warm to the touch (but contains a controlled fire inside) you are witnessing an example of thermal conduction (heat transfer across the wall due to molecular activity). A room in the summer time gains heat through the walls through conduction.

Consider a wall of thickness L with one side of the wall at a temperature T_1 and the other side at a temperature T_2 , as shown in Figure 1-1. If T_1 and T_2 don't change with time, we have a steady-state. If the wall is much taller and wider than it is thick, then the direction of heat flow will solely be along the x axis of Figure 1-1. Therefore, the heat transfer is one-dimensional.

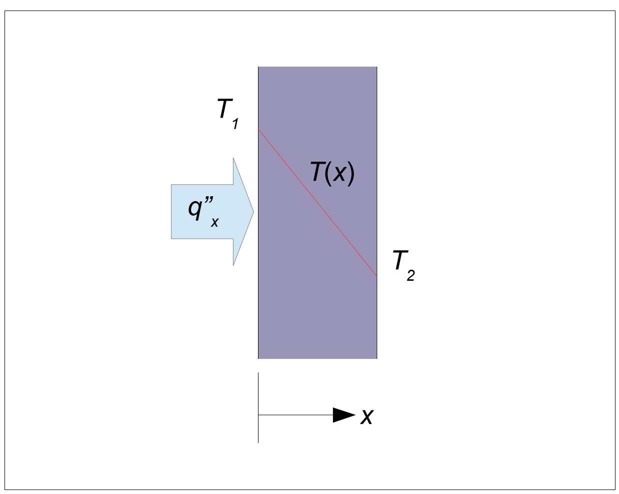


Figure 1-1. One-dimensional, steady conduction through a wall

The rate of heat transfer by conduction is given by **Fourier's law**,

$$q_x" = -k \frac{dT}{dx} \tag{1-1}$$

The heat flux $q_x"$ is the heat transfer rate in the x-direction per unit area perpendicular to the direction of transfer, and it is proportional to the temperature gradient, dT/dx , in this direction. The parameter k is the thermal conductivity and is a characteristic of the wall material. Heat is transferred in the direction of decreasing temperature, hence the negative sign in Equation 1-1. Under the steady-state conditions shown in Figure 1-1, where the temperature distribution is linear, the temperature gradient is:

$$\frac{dT}{dx}=\frac{T_2-T_1}{L}$$

so the heat flux is:

$$q_x" = k\frac{T_1-T_2}{L} = k\frac{\Delta T}{L} \qquad (1\text{-}2)$$

Note that Equation 1-2 provides a heat flux, or heat transfer rate per unit area perpendicular to the flow direction. The heat rate by conduction, q_x (Btu/hour or W), through a plane wall of area A is the product of the flux and the area: $q_x=q_x"A$.

In SI units, heat flux is in W/m^2. In US Customary System, heat flux is typically in (Btu/hour)/ft^2 or (Btu/hour)/in^2. Similarly, the thermal conductivity is in W/(m·°C) in SI units, and in (Btu/hour)/(ft·°F) something equivalent such as (Btu·in/hour)/(ft^2·°F). You might encounter W/(m·K) or (Btu/hour)/(ft·°R) that is, with the absolute temperature units in the denominator. This is inconsequential because (Btu/hour)/(ft·°R) and (Btu/hour)/(ft·°F) are completely interchangeable without a need to convert between them. The reason is that thermal conductivity is always multiplied by a temperature difference (as in Equation 1-2) so a ΔT of, say, 25°F is also a ΔT of 25°R.

For the purposes of the PE exam, we are especially interested in convection heat transfer, when it occurs between a fluid in motion and a bounding surface when the two are at different temperatures. Convection heat transfer may be classified according to the nature of the flow.

Forced convection occurs when the flow is caused by external means, such as by a fan or pump. As an example, consider the use of a fan to provide forced convection air cooling of hot tubes and fin in an automotive radiator. In contrast, for **free (or natural) convection**, the flow is induced by buoyancy forces, which are due to density differences caused by temperature variations in the fluid. An example is the free convection heat transfer that occurs from hot metal parts removed from a heat treatment furnace and placed in a quiescent pool of cool liquid for quenching. The liquid that makes contact with the metal parts experiences an increase in temperature and hence a reduction in density. Since it is now lighter than the surrounding liquid, buoyancy forces induce a vertical motion for which warm liquid ascending from the metal parts is replaced by an inflow of cooler liquid.

There are situations in which **mixed convection** exists, which is when the effects of forced and natural convection are comparable. An important subset of convection problems involve **boiling and condensation**. In these situations, the effects of phase-change and the transfer of latent heat are dominant.

Independent of the nature of the convection heat transfer process, the appropriate rate equation is:

$$q'' = h\left(T_s - T_\infty\right) \tag{1-3}$$

where q'', the convective heat flux is proportional to the difference between the surface and fluid temperatures, T_s and T_∞ respectively. This expression is known as Newton's law of cooling, and the parameter h is the convection heat transfer coefficient, or film coefficient. This coefficient depends on conditions in the boundary layer, which are influenced by surface geometry, the nature of the fluid motion, and an assortment of fluid thermodynamic and transport properties.

For a fixed heat flux q'', a small h means a high temperature difference between the surface and the bathing fluid. Conversely, a high h means a low temperature difference between the surface and the bathing fluid. You might see in a problem statement a phrase such as: "*a very large heat transfer coefficient*", which means that it is safe to assume that the surface and the bathing fluid are practically at the same temperature. Convection heat transfer analysis generally consists of determining the convection coefficient.

The units of h in SI are typically W/(m^2·°C) and Btu/(hour·ft^2·°F) in the US Customary System. You might encounter W/(m^2·K) or (Btu/hour)/(ft^2·°R) that is, with the absolute temperature units in the denominator. This is inconsequential because (Btu/hour)/(ft^2·°R) and (Btu/hour)/(ft^2·°F) are completely interchangeable without a need to convert between them. The reason is that the heat transfer coefficient is always multiplied by a <u>temperature difference</u> (as in Equation 1-3) so a ΔT of, say, 25°F is also a ΔT of 25°R.

Thermal radiation is energy emitted by matter that is at a nonzero absolute temperature. The energy is transported by electromagnetic waves (or alternatively, photons). While the transfer of energy by conduction or convection requires the presence of a material medium, radiation does not. The energy from the sun that reaches Earth's atmosphere does so by radiation.

A surface at an absolute temperature T_s will emit energy at a certain rate per unit area, known as the emissive power E. A theoretically maximum emissive power E_b is given by the **Stefan-Boltzmann law**:

$$E_b = \sigma T_s^4 \tag{1-5}$$

where $\sigma = 5.67\times10^{-8}\,\mathrm{W/(m^2 \cdot K^4)} = 1.714\times10^{-9}\,\mathrm{(Btu/h)/(ft^2\,°R^4)}$ is the Stefan-Boltzmann constant. A surface for which the emissive power is given by Equation 1-5 is known as a **blackbody**, or an **ideal radiator**.

The emissive power of a real surface is less than that of the ideal radiator and is given by:

$$E = \epsilon\, \sigma T_s^4 \tag{1-6}$$

where ϵ is the emissivity, a property that depends on the surface finish and material. The emissivity is always less than one, so it provides an efficiency of thermal radiation emission with respect to that of a blackbody.

Radiation may also be incident on, (or received by) a surface. Incident radiation on a surface may come from the sun, a flame, a fireball, or other surfaces to which the surface is exposed to. The rate at which all such radiation is incident on a unit area of the surface as the irradiation G.

The rate at which irradiation is absorbed may be evaluated from knowledge of a surface radiative property termed the absorptivity α as follows:

$$G_{\mathrm{abs}} = \alpha\, G \tag{1-7}$$

where $0 \leqslant \alpha \leqslant 1$. A surface for which $\alpha = \epsilon$ is called a **gray surface**. In addition to being absorbed, irradiation may be reflected (by opaque surfaces) and transmitted (by semitransparent surfaces). The value of α depends on the nature of the irradiation, as well as on the surface itself. For example, the absorptivity of a surface to solar radiation may differ from its absorptivity to radiation emitted by the walls of a furnace or by other heat sources.

A common case occurs when a small surface at absolute temperature T_s is completely surrounded by a much bigger isothermal surface (which we call "the surroundings") at T_∞. It can be shown that the incident radiation on the small surface is $G = \sigma T_\infty^4$. If the emissive power for the small surface is $E = \epsilon E_b(T_s)$, then the net thermal radiation from the surface to the surroundings is $q''_{\text{rad}} = E - \alpha G$. If we assume the small surface is gray, then:

$$q''_{\text{rad}} = \epsilon \sigma \left(T_s^4 - T_\infty^4\right) \tag{1-8}$$

It is convenient sometimes to linearize Equation 1-6 as:

$$q''_{\text{rad}} = h_{\text{rad}} \left(T_s - T_\infty\right) \tag{1-9}$$

where h_{rad} is the radiation heat transfer coefficient defined as:

$$h_{\text{rad}} = \epsilon \sigma \left(T_s + T_\infty\right)\left(T_s^2 + T_\infty^2\right) \tag{1-10}$$

In heat transfer problems, it is common to apply an energy balance on a surface. The control volume surfaces are located on either side of the physical boundary. Such control volumes enclose no mass, so energy can't be stored in these control volumes. An example is shown in Figure 1-2.

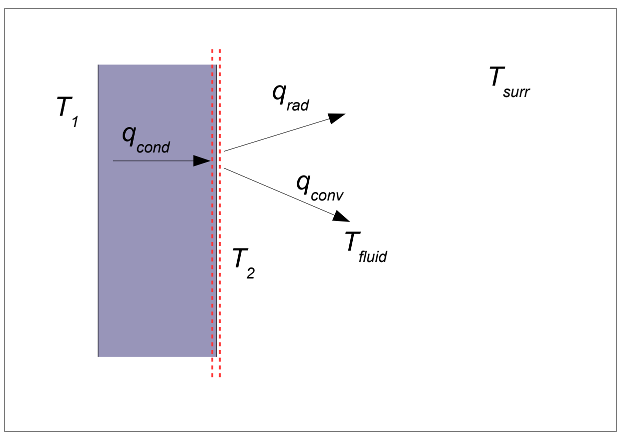


Figure 1-2. Surface energy balance

The figure shows a wall that is subject to a temperature gradient across its thickness. The left side is at a temperature T_1 and the right side at T_2. This gradient drives a conductive heat transfer across the solid wall. The right side is bathed by a fluid of temperature T_{fluid}, which drives a convective heat transfer from the right wall surface to the fluid. Finally, the surroundings are at T_{surr} therefore, a net radiative transfer exists between the right side of the wall and the surroundings. The surface control volume is indicated with dashed red lines. The rate at which energy enters the control volume must equal the rate at which it leaves, so for the example of Figure 1-2, the surface energy balance is:

$$q''_{\text{cond}} = q''_{\text{rad}} + q''_{\text{conv}}$$

PROBLEMS

01-01. The wall of an industrial furnace is 6-in thick and built from a material with a thermal conductivity of 1.0 (Btu/h)(ft·°F). Steady-state operating temperatures of 2060 °F and 1610 °F are recorded at the inner and outer surfaces, respectively. What is the rate of heat loss through a wall that is 20 in × 48 in on a side?

01-02. A heat transfer rate of 5 kW is conducted through a section of an insulating material of cross-sectional area 10 m^2 and thickness 2.5 cm. If the inner (hot) surface temperature is 375°C and the thermal conductivity of the material is 0.12 W/(m·K), what is the outer surface temperature?

01-03. A laboratory hot-plate heater with a diameter of 6 inches is left on and it reaches a steady-state temperature of 500°F at the surface. The power delivered from the heating element to the plate is 340 Btu/hour. The average bulk temperature of the air surrounding the plate is approximately 85°F. Assume all heat transfer from the plate occurs by convection only. Calculate the convective coefficient.

01-04*. A freezer compartment consists of a cubical cavity that is 6.5 ft on a side with the bottom side perfectly insulated. The normal operating temperatures of the inner and outer surfaces are 15°F and 95°F, respectively. Under these conditions, the thickness of styrofoam insulation $\left(k = 0.017\,\mathrm{Btu/h/ft/°F}\right)$ that must be applied to the top and side walls to ensure a heat load of 1,700 Btu/h, is most nearly:

(A) 0.17

(B) 1.5

(C) 2

(D) 2.5

01-05*. The wall of an industrial heat treatment oven is 6-in thick and composed of fireclay brick (thermal conductivity of 1 (Btu/h)/(ft·°F)). The inner face is subject to a heat flux of 35 (Btu/h)/ft^2. The opposite face is exposed to air at 85°F with a convection coefficient of 3.5 (Btu/h)/(ft^2·°F). Under steady state operation, the temperature (°F) of the wall surface exposed to air is most nearly:

(A) 90

(B) 95

(C) 100

(D) 105

01-06*. The wall of an industrial heat treatment oven is 8-in thick and composed of fireclay brick (thermal conductivity of 0.93 (Btu/h)/(ft·°F)). The inner face is subject to a heat flux of 25 (Btu/h)/ft^2. The opposite face is exposed to air at 75°F with a convection coefficient of 3.5 (Btu/h)/(ft^2·°F). Under steady state operation, the temperature (°F) of the inner wall surface air is most nearly:

(A) 82

(B) 96

(C) 100

(D) 297

01-07*. A wall has inner and outer surface temperatures of 15°C and 6°C, respectively. The interior and exterior air temperatures are 20°C and 5°C, respectively. The outer convection heat transfer coefficient is 20 W/(m^2K). The inner convection heat transfer coefficient (W/(m^2K)) is most nearly:

(A) 4

(B) 10

(C) 50

(D) 100

01-08*. An electric resistance heater is embedded in a 3 feet long cylinder of 1¼ inch diameter. When water with a temperature of 77°F and velocity of 3.5 ft/s flows crosswise over the cylinder, the power consumed by the heater to maintain the surface at a uniform temperature of 195°F is 25 kW. Under these conditions, the heat transfer coefficient ((Btu/h)/(ft^2·°F)) for the water bathed surface of the cylinder is most nearly:

(A) 190

(B) 370

(C) 520

(D) 740

01-09*. An automotive transfer case transfers power from the transmission to the rear axles with an efficiency of 90%. The transfer case can be modeled as a cube of 1 ft length, cooled by 85°F air and a convective coefficient of 35 (Btu/h)/(ft^2·°F) on all sides. When the power from the transmission is 150 hp, the average surface temperature (°F) of the transfer case is most nearly:

(A) 85

(B) 267

(C) 295

(D) 1,200

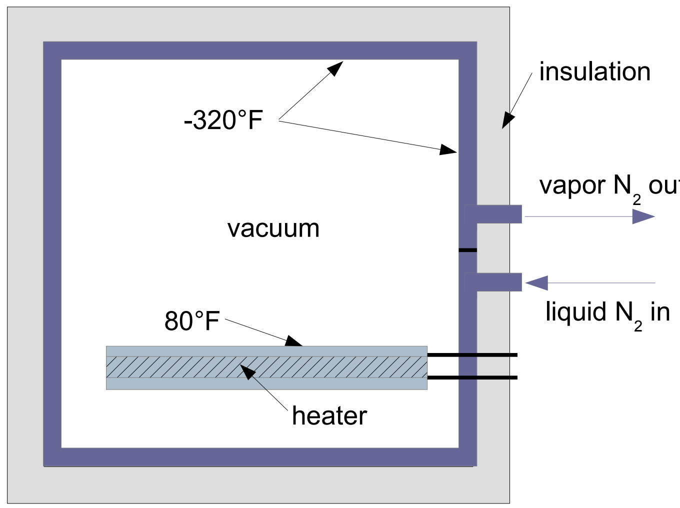
01-10*. The manufacturing process of thin films on microcircuits uses a perfectly insulated vacuum chamber whose walls are kept at -320°F by a liquid nitrogen bath. An electric resistance heater is embedded inside a 1.5 feet long cylinder of 1½ inch diameter placed inside the vacuum chamber. The surface of the cylinder has an emissivity of 0.25 and is maintained at 80°F by the heater. The nitrogen enters the chamber bath as a saturated liquid and leaves as a saturated vapor. Neglecting any heat transfer from the ends of the cylinder, the required flow rate of nitrogen (pounds-mass per hour) is most nearly (the latent heat of vaporization for N_2 is 53.74 Btu per pound):

(A) 0.05

(B) 0.1

(C) 0.2

(D) 0.4



01-11*. A laboratory hot-plate heater with a diameter of 6 inches and an emissivity of 0.75 is left on and it reaches a steady-state temperature of 500°F at the surface. The average bulk temperature of the air surrounding the plate is approximately 100°F, the convection heat transfer coefficient is 2.5 (Btu/h)/(ft^2 °F) and the walls and ceiling of the lab are at 65°F. Under these conditions, the total heat transfer rate (Btu per hour) from the hot plate is most nearly:

(A) 212

(B) 392

(C) 1,994

(D) 56,384

Solutions:

01-01. This is a straightforward application of Equation 1-2 to get the heat flux:

$$q_x"=k\frac{\Delta T}{L} = 1.0\frac{\text{Btu/h}}{\text{ft °F}}\times\frac{(2{,}060-1{,}610)\text{°F}}{6\text{ in}\times\left|\frac{1\text{ ft}}{12\text{ in}}\right|} = 900\frac{\text{Btu/h}}{\text{ft}^2}$$

So the heat transfer rate is:

$$q_x = q_x"A = 900\frac{\text{Btu/h}}{\text{ft}^2}\times(20\times48)\text{in}^2\times\left|\frac{1\text{ ft}^2}{144\text{ in}^2}\right| = 6{,}000\text{ Btu/h}$$

01-02. The heat flux is the heat transfer rate divided by the cross sectional area:

$$q_x" = \frac{q_x}{A} = \frac{5\times10^3\text{ W}}{10\text{ m}^2} = 500\frac{\text{W}}{\text{m}^2}$$

We can solve for the low temperature T_2 from Equation 1-2:

$$T_2 = T_1 - \frac{q_x"L}{k} = 375\text{°C} - \frac{500\frac{\text{W}}{\text{m}^2}\times0.025\text{ m}}{0.12\frac{\text{W}}{\text{m}\cdot\text{K}}} = 271\text{°C}$$

01-03. We can solve for the convection coefficient from Equation 1-3, Newton's law of cooling, keeping in mind that the heat flux is the heat transfer rate per unit surface area:

$$h = \frac{q"}{(T_s-T_\infty)} = \frac{\left(\frac{340\text{ Btu/h}}{\frac{\pi}{4}(6\text{ in})^2}\right)}{(500-85)\text{°F}} = 0.02898\frac{\text{Btu/h}}{\text{in}^2\text{°F}} = 4.18\frac{\text{Btu/h}}{\text{ft}^2\text{°F}}$$

01-04*. The correct answer is (C).

The thickness of the insulation can be obtained from Fourier's law, Equation 1-2 $L=k\Delta T/q_x"$**,** where the heat flux is the heat transfer rate divided by the total area:

$$q_x" = \frac{q_x}{A} = \frac{1{,}700\text{ Btu/h}}{5\times(6.5\text{ ft}\times6.5\text{ ft})} = 8.05\frac{\text{Btu/h}}{\text{ft}^2}$$

Therefore:

$$L = k\frac{\Delta T}{q_x"} = 0.017\frac{\text{Btu/h}}{\text{ft °F}}\times\frac{(95-15)\text{°F}}{8.05\frac{\text{Btu/h}}{\text{ft}^2}} = 0.169\text{ ft} \approx 2\text{ in}$$

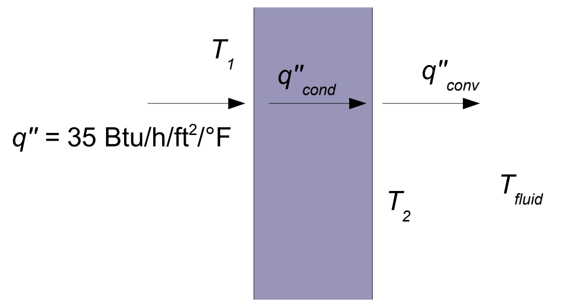
01-05*. The correct answer is (B).

At steady-state, the energy (heat flux) that enters the wall on the left side must be conducted through the wall and then leave by convection from the right side at the same rate. Therefore, we can solve for the wall temperature from Newton's law of cooling:

$$q''_{\text{conv}} = h\left(T_2 - T_{\text{fluid}}\right)$$

$$\Rightarrow T_2 = T_{\text{fluid}} + \frac{q''_{\text{conv}}}{h}$$

$$T_2 = 85\,°\text{F} + \frac{35\,\dfrac{\text{Btu/h}}{\text{ft}^2}}{3.5\,\dfrac{\text{Btu/h}}{\text{ft}^2\,°\text{F}}} = 95\,°\text{F}$$



01-06*.

This is the same situation as the previous problem, but we are now asked to find T_1. At steady-state, the energy (heat flux) that enters the wall on the left side must be conducted through the wall at the same rate. Therefore:

$$q''_{\text{cond}} = -k\frac{\left(T_1 - T_2\right)}{L} \quad \Rightarrow \quad T_1 = T_2 + \frac{q''_{\text{cond}}L}{k}$$

and we obtain T_2 from an analysis like that of the previous problem:

$$T_2 = T_{\text{fluid}} + \frac{q''_{\text{conv}}}{h} \quad \Rightarrow \quad T_2 = 75\,°\text{F} + \frac{25\,\dfrac{\text{Btu/h}}{\text{ft}^2}}{3.5\,\dfrac{\text{Btu/h}}{\text{ft}^2\,°\text{F}}} = 82.1\,°\text{F}$$

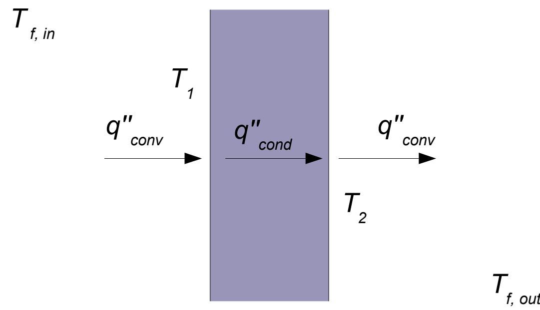
Hence:

$$T_1 = T_2 + \frac{q''_{\text{cond}}L}{k} = 82.1\,°\text{F} + \frac{25\,\dfrac{\text{Btu/h}}{\text{ft}^2} \times 8\,\text{in} \times \left|\dfrac{1\,\text{ft}}{12\,\text{in}}\right|}{0.93\,\dfrac{\text{Btu/h}}{\text{ft}\,°\text{F}}} \approx 100\,°\text{F}$$

01-07*. The correct answer is (A).

At steady-state, the energy (heat flux) that enters the wall on the left side (by convection) must be conducted through the wall and then leave (by convection) from the right side at the same rate:

$$q''_{\text{conv}} = h_{\text{in}}\left(T_{\text{f, in}} - T_1\right)$$
$$q''_{\text{conv}} = h_{\text{out}}\left(T_2 - T_{\text{f, out}}\right)$$



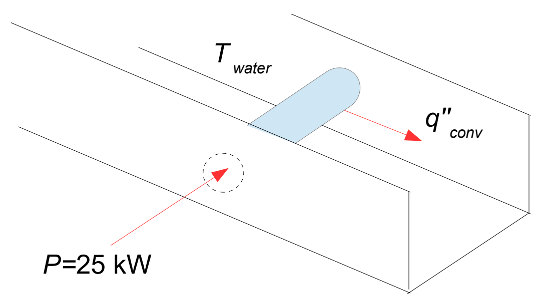
We can equate these two quantities:

$$h_{\text{in}}\left(T_{\text{f, in}} - T_1\right) = h_{\text{out}}\left(T_2 - T_{\text{f, out}}\right)$$
$$h_{\text{in}} = h_{\text{out}}\left(\frac{T_2 - T_{\text{f, out}}}{T_{\text{f, in}} - T_1}\right)$$
$$h_{\text{in}} = 20\,\frac{\text{W}}{\text{m K}} \times \left(\frac{6-5}{20-15}\right) = 4\,\frac{\text{W}}{\text{m K}}$$

01-08*. The correct answer is (D)

From Newton's law of cooling we can solve for h :

$$q''_{\text{conv}} = h\left(T_s - T_{\text{water}}\right)$$
$$\Rightarrow h = \frac{q''_{\text{conv}}}{\left(T_s - T_{\text{water}}\right)}$$



and an energy balance on the heater shows that, for steady-state, the power P delivered to the heater must equal the energy convected away by the water:

$$P = \left(q''_{\text{conv}}\right) A$$

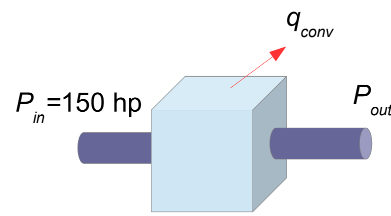
where A is the surface area of the cylinder bathed by the water. Hence

$$q''_{\text{conv}} = \frac{P}{\pi D L} = \frac{25\,\text{kW} \times \left|\frac{3{,}412\,\text{Btu/h}}{1\,\text{kW}}\right|}{\pi \times 1.25\,\text{in} \times \left|\frac{1\,\text{ft}}{12\,\text{in}}\right| \times 3\,\text{ft}} = 86{,}886\,\frac{\text{Btu/h}}{\text{ft}^2}$$

Now, we can calculate h

$$h = \frac{q''_{\text{conv}}}{\left(T_s - T_{\text{water}}\right)} = \frac{86{,}886\,\frac{\text{Btu/h}}{\text{ft}^2}}{(195-77)\,°\text{F}} \approx 736\,\frac{\text{Btu/h}}{\text{ft}^2\,°\text{F}}$$

01-09*. The correct answer is (B)



Since the efficiency of the transfer case is 90%, then 10% of the power input is dissipated as stray heat, which is convected away by the air. Therefore:

$$q_{\text{conv}} = 0.1 \times 150\text{ hp} \times \left| \frac{2{,}544.43\text{ Btu/h}}{1\text{ hp}} \right| = 38{,}166.45\text{ Btu/h}$$

So, the heat flux is:

$$q''_{\text{conv}} = \frac{P}{A} = \frac{38{,}166.45\text{ Btu/h}}{6 \times 1\text{ ft} \times 1\text{ ft}} = 86{,}886\frac{\text{Btu/h}}{\text{ft}^2} = 6{,}361\frac{\text{Btu/h}}{\text{ft}^2}$$

Now, we can solve for the surface temperature:

$$q''_{\text{conv}} = h\left(T_s - T_{\text{air}}\right)$$

$$\Rightarrow T_s = T_{air} + \frac{q''_{\text{conv}}}{h} = 85\,°\text{F} + \frac{6{,}361\dfrac{\text{Btu/h}}{\text{ft}^2}}{35\dfrac{\text{Btu/h}}{\text{ft}^2\,°\text{F}}} \approx 267\,°\text{F}$$

01-10*.

The correct answer is (D).

An energy balance on the nitrogen, shows that the heat added to the nitrogen must be equal to the nitrogen flow rate times the enthalpy change across the bath:

$$q = \dot{m}_{\text{N}_2} h_{fg} \;\Rightarrow\; \dot{m}_{\text{N}_2} = q/h_{fg}$$

The heat added to the nitrogen comes from the warm surface of the cylinder, via radiation. The heat transfer from the cylinder is:

$$q = A\,q''_{\text{rad}} = \left(\pi\, D\, L\right)\epsilon\,\sigma\left(T_s^4 - T_{\text{walls}}^4\right)$$

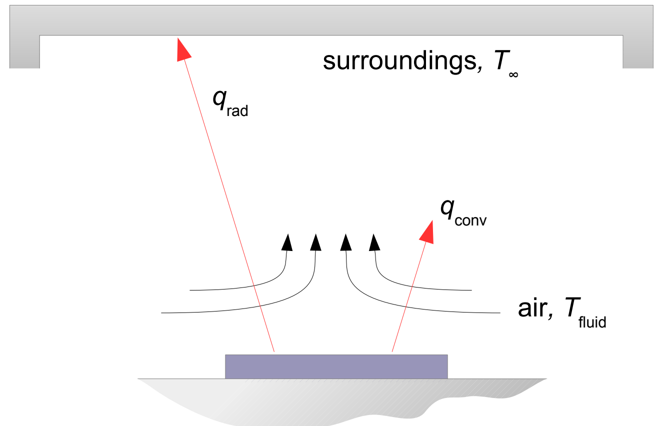
$$= \left(\pi \times 1.5\text{ in} \times \left|\frac{1\text{ ft}}{12\text{ in}}\right| \times 1.5\text{ ft}\right) 0.25 \times 1.714 \times 10^{-9}\frac{\text{Btu/h}}{\text{ft}^2\,°\text{R}^4} \times \left(540^4 - 140^4\right)°\text{R}^4 = 21.37\text{ Btu/h}$$

Hence, the mass flow of nitrogen is:

$$\dot{m}_{\text{N}_2} = \frac{q}{h_{fg}} = \frac{21.37\text{ Btu/h}}{53.74\text{ Btu/lbm}} = 0.398\text{ lbm/h}$$

01-11*. The correct answer is (A)

The total heat transfer from the hot plate is the sum of the convective and radiative heat transfer rates.



$$q''_{\text{total}} = q''_{\text{rad}} + q''_{\text{conv}}$$

where:

$$q''_{\text{rad}} = \epsilon\,\sigma\left(T_s^4 - T_\infty^4\right)$$

and

$$q''_{\text{conv}} = h\left(T_s - T_\infty\right)$$

The radiation calculation must be performed with absolute temperatures, so we convert $T_s = 500\,°\text{F} = 960\,°\text{R}$ and $T_\infty = 65\,°\text{F} = 525\,°\text{R}$

$$q''_{\text{rad}} = \epsilon\,\sigma\left(T_s^4 - T_\infty^4\right) = 0.75 \times 1.714 \times 10^{-9}\frac{\text{Btu/h}}{\text{ft}^2\,°\text{R}^4} \times \left(960^4 - 525^4\right)°\text{R}^4 = 994.2\frac{\text{Btu/h}}{\text{ft}^2}$$

Now, calculate the convective flux:

$$q''_{\text{conv}} = h\left(T_s - T_{\text{fluid}}\right) = 2.5\frac{\text{Btu/h}}{\text{ft}^2\,°\text{F}} \times (500 - 100)\,°\text{F} = 1{,}000\frac{\text{Btu/h}}{\text{ft}^2}$$

Hence

$$q_{total} = \frac{\pi}{4}\left(6\,\text{in}^2\right) \times \left|\frac{1\,\text{ft}^2}{144\,\text{in}^2}\right| \times \left(994.2 + 1{,}000\right)\frac{\text{Btu/h}}{\text{ft}^2} = 216\,\text{Btu/h}$$

02: 1D Steady State Conduction – The Plane Wall

In this section we continue the discussion on heat conduction and introduce some advanced concepts, but still staying within the context of one-dimensional, steady state problems.

<u>The Plane Wall</u>

Consider the wall of thickness L and thermal conductivity k Figure 2-1. This wall separates a hot fluid at $T_{\infty,1}$ from a cold fluid at $T_{\infty,2}$. It can be shown that the temperature variation *within the wall* is linear from a value T_1 (for the surface in contact with the hot fluid) to a value T_2 (for the surface in contact with the cold fluid).

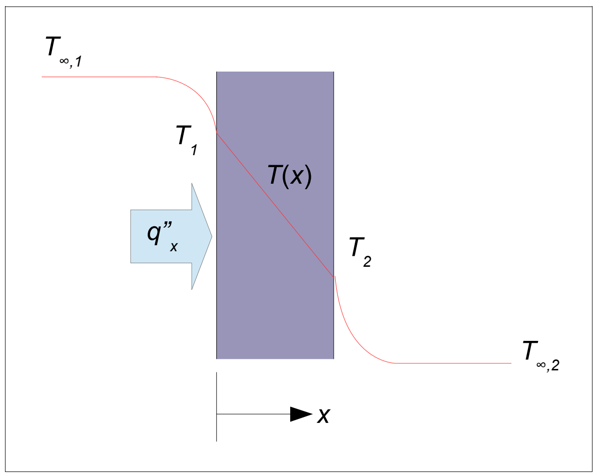


Figure 2-1. Plane wall with convective heat transfer on both sides

From Fourier's law, in the situation of Figure 2-1, the heat transfer rate is

$$q_x = \frac{k\,A}{L}(T_1 - T_2) \tag{2-1}$$

where A is the wall area perpendicular to the direction of the flow of heat.

Equation 2-1 indicates that – for a plane wall – the heat transfer rate is a constant, independent of x. The temperature difference $\Delta T = (T_1 - T_2)$ can be interpreted as a "driving potential" that triggers the transfer of heat. Whenever $\Delta T \neq 0$ there will be heat transfer. For a fixed value of $\Delta T \neq 0$, the magnitude of the heat transfer can be varied with the magnitude of the parameter $L/(k\,A)$, which is known as the **thermal resistance for conduction**, $R_{t,\text{cond}}$:

$$R_{th,\,\text{cond}} = \frac{L}{k\,A} \tag{2-2}$$

So that Equation 2-1 can be written as $q_x = \Delta T / R_{th,\text{cond}}$. For a fixed temperature change, the higher the thermal resistance, the lower the heat transfer rate and vice-versa. Note that the thermal resistance decreases with increasing conductivity and with decreasing thickness. Therefore, thin layers of highly conducting materials (for example thin walled metal pipes) experience very small temperature gradients, for

The concept of thermal resistance can be developed for convection as well. Consider the right side of the wall in Figure 2-1. The heat flux can be expressed as $q_x = (T_2 - T_{\infty,2}) / R_{th,\text{conv}}$ where the **thermal resistance for convection**, is defined as:

$$R_{th,\,\text{conv}} = \frac{1}{h\,A} \tag{2-3}$$

A t**hermal resistance for radiation** is defined the same way as in Equation 2-3 but the coefficient h is the radiation coefficient we defined in Equation 1-10.

In Figure 2-1 we see that the heat is being transferred from the hot fluid to the cold fluid, because there is a non-zero, overall temperature difference, $T_{\infty,1} - T_{\infty,2}$. In order to do this, the heat has to "make it through" three resistances: a convective resistance on the left side, a conductive resistance, and another convective resistance at the right side. This is analogous to the flow of current due to a voltage potential across resistors. The thermal resistances in the example of Figure 2-1 are in series, and we can develop the analogous "thermal circuit" of Figure 2-2:

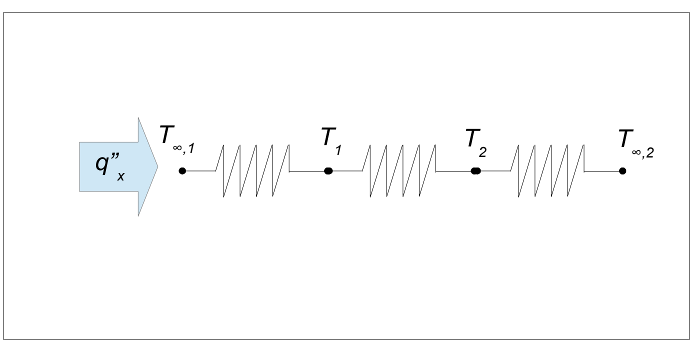


Figure 2-2. "Thermal Circuit" for Plane wall with convective heat transfer on both sides of Figure 2-1

So, just like in the analysis of circuits, we have multiple expressions for the heat transfer rate (current intensity):

$$q_x = \frac{T_{\infty,1} - T_1}{1/(h_1 A)} = \frac{T_1 - T_2}{L/(k A)} = \frac{T_2 - T_{\infty,2}}{1/(h_2 A)} \quad (2\text{-}4)$$

and, like in the analysis of electrical circuits, it can be shown that the three resistance in series can be added when defining a total resistance R_{th} , so that:

$$q_x = \frac{T_{\infty,1} - T_{\infty,2}}{R_{th}} \quad (2\text{-}5)$$

where the total resistance for the example of Figure 2-1 would be defined as:

$$R_{th} = \frac{1}{h_1 A} + \frac{L}{k A} + \frac{1}{h_2 A} \quad (2\text{-}6)$$

We may also use equivalent thermal circuits for more complex systems, such as **composite walls**. Such walls may involve any number of series and parallel thermal resistances due to layers of different materials. Consider the series composite wall of Figure 2-3:

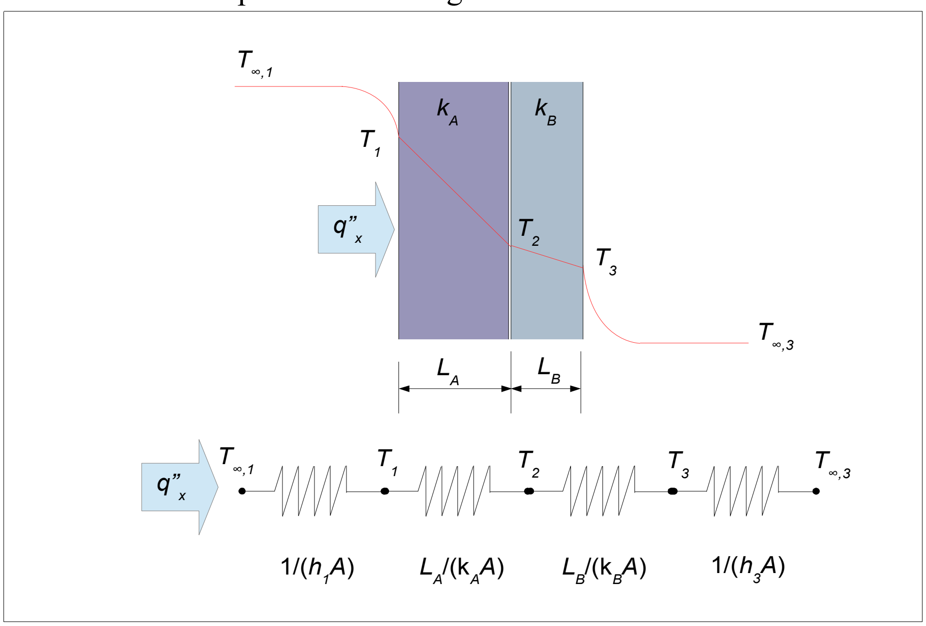


Figure 2-3. Series composite wall, and equivalent resistances

We can write the heat transfer rate in terms of the overall temperature difference:

$$q_x = \frac{T_{\infty,1} - T_{\infty,3}}{1/(h_1 A) + L_A/(k_A A) + L_B/(k_B A) + 1/(h_3 A)} \quad (2\text{-}7)$$

where the group of terms in the denominator comprise the total resistance. We can also express the heat transfer rate in terms of the individual elements,

$$q_x = \frac{T_{\infty,1}-T_q}{1/(h_1 A)} = \frac{T_1-T_2}{L_A/(k_A A)} = \frac{T_2-T_3}{L_B/(k_B A)} = \frac{T_3-T_{\infty,3}}{1/(h_3 A)}$$

or in terms of partial components of the entire wall, for example,

$$q_x = \frac{T_{\infty,1}-T_2}{1/(h_1 A)+L_A/(k_A A)}$$

which can be used to determine temperatures at the interfaces between components (like T_2, for example).

Resistances can be also arranged in parallel, such as in the situation shown in Figure 2-4. In this case, the radiative and convective resistances are in parallel.

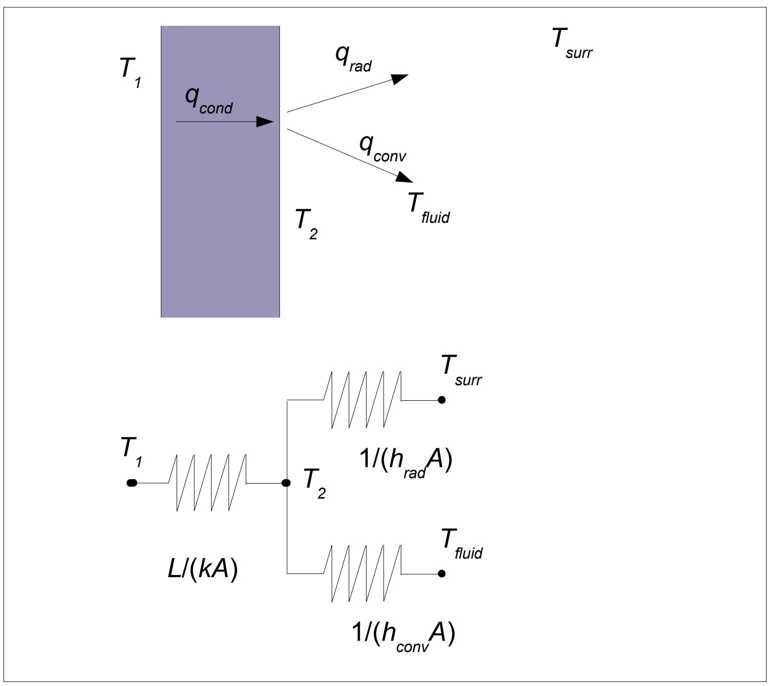


Figure 2-4. Resistances in parallel

If the fluid (for convection) and the surroundings (for radiation) are at the same temperature, $T_{\text{fluid}} = T_{\text{surr}} = T_{\infty}$ then the circuit analogy can be used to develop an expression for the equivalent resistance from convection and radiation. For the example of Figure 2-4 the resistances in parallel are combined as:

$$\frac{1}{R_{th\,\text{conv,rad}}}=\left(\frac{1}{h_{\text{conv}}A}\right)^{-1}+\left(\frac{1}{h_{\text{rad}}A}\right)^{-1}$$
$$\Rightarrow R_{th,\,\text{conv,rad}}=\frac{1}{A\left(h_{\text{conv}}+h_{\text{rad}}\right)}$$

The heat flux expressed in terms of the overall temperature difference would be:

$$q_x=\frac{T_1-T_\infty}{R_{th,\,\text{total}}}=\frac{T_1-T_\infty}{R_{th,\,\text{cond}}+R_{th,\,\text{conv \& rad}}}=\frac{T_1-T_\infty}{R_{th,\,\text{cond}}+R_{th,\,\text{conv \& rad}}}=\frac{T_1-T_\infty}{\frac{1}{A}\left(\frac{L}{k}+\frac{1}{h_{\text{conv}}+h_{\text{rad}}}\right)}$$

Up to now, we have assumed that there is one unique value of temperature at the interface between two layers in a composite wall. For example, the right side of layer "A" and the left side of layer "B" in Figure 2-3 are both at T_2. In reality, the surfaces in contact have some roughness associated with it and there are small air gaps and pockets. These air gaps represent another resistance so the mating sides in "real-world" interfaces will be at different temperatures. We model this as a discontinuity in the temperature profile, as shown in Figure 2-5:

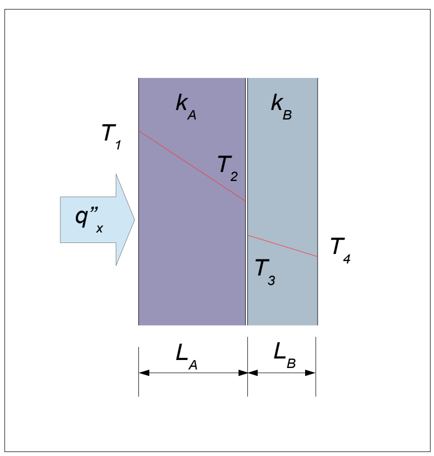


Figure 2-5. The effect of non-negligible contact resistance.

In the figure we see that the temperature drops from T_2 to T_3 because of the contact resistance. The contact resistance is thus defined as:

$$R_{th,\,\text{contact}}=\frac{\left(T_2-T_3\right)}{q''_x} \tag{2-8}$$

Values of contact resistance (per unit area normal to heat flow) are listed in the literature as a function of the materials of the layers, the contact pressure, and the interstitial fluid between layers.

Sometimes (especially in the construction industry) you will encounter the term "**R-value**". The R-value of a layer of material is the thermal resistance per unit area. Typical units are (°F ft^2)/(Btu/h). The R-value is not the same as the thermal resistance we have been discussing so far. They are however, related by:

$$\text{R-value} = R_{th} A = \frac{\Delta T}{q_x''} \tag{2-9}$$

Given the definition of R-value, and that of thermal resistance for convection, it can be seen that the R-value for a layer of fluid is the inverse of the convection coefficient. Also for flat walls in which the area remains constant, the heat flux q_x'' is also constant, so R-values in series can be added to find equivalent R-values.

With composite systems, it is often convenient to work with an **overall heat transfer coefficient** U , also known as the **U-factor** which is defined by:

$$q_x = UA\,\Delta T \tag{2-10}$$

where ΔT is the overall temperature difference and, for the example of Figure 2-3:

$$U = \frac{1}{R_{th} A} = \frac{1}{1/h_1 + L_A/k_A + L_B/k_B + 1/h_3} \tag{2-11}$$

Note that the U-factor is the inverse of the R-value.

PROBLEMS

02-01. The wall of an industrial furnace is built from 6-in thick fireclay brick and it is covered with a 1/4-inch gypsum board layer. Steady-state operating temperatures of 2,060 °F and 100 °F are recorded at the inner and outer surfaces, as shown in the sketch. Neglecting any thermal radiation, calculate:

a) The temperature (°F) at the brick-gypsum interface.

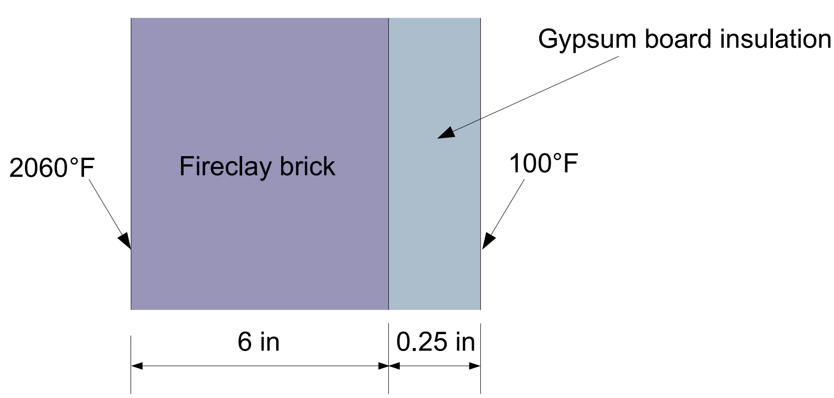
b) The heat flux (Btu/h)/(ft^2) and the heat transfer rate (Btu/h) across the wall, if its dimensions are 6 ft × 6 ft.

c) The R-value and U-factor for the composite wall.

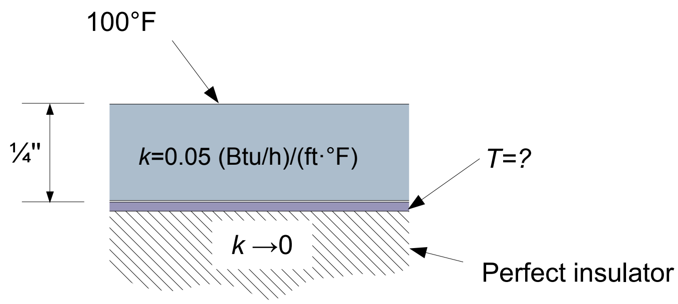
d) The air room temperature on the gypsum board side if the convective coefficient at the exposed side of the gypsum board insulation is 35 (Btu/h)/(ft^2·°F).

e) The convective coefficient on the exposed side of the fireclay brick wall, if the average temperature of the gases inside the furnace is 2150°F.

	Thermal Conductivity (Btu/h)/(ft·°F)
Fireclay brick	0.25
Gypsum board	0.09

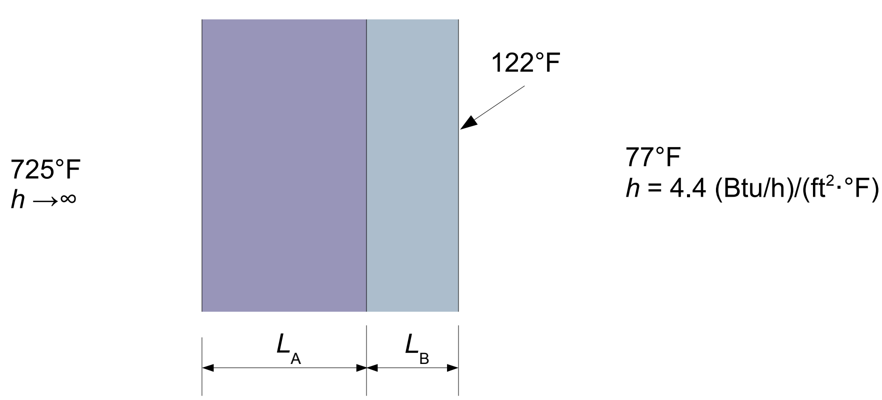


02-02. A thin, highly-conducting metal heater generates heat at a rate of 650 (Btu/h)/ft^2. The heater is placed on a non-conducting substrate (a perfect insulator). There is a 1/4-inch thick layer of a material with a conductivity of 0.05 (Btu/h)/(ft·°F) on top of the heater. The top surface of this layer is kept at 100°F. Find the temperature of the heater.



02-03*. A self-cleaning residential convection oven uses a composite window. The window consists of two high-temperature plastics (A and B) of thicknesses $L_A = 2\,L_B$ and thermal conductivities k_A of 0.087 (Btu/h)/(ft·°F) and k_B of 0.046 (Btu/h)/(ft·°F). During steady state operation at the highest setting, the air inside the oven is 725°F, while the room air temperature is 77°F. The inside convection coefficient is infinitely large and the outside convection coefficient is 4.4 (Btu/h)/(ft^2·°F). To avoid a skin contact burn hazard, the outer surface window shall not exceed 122°F. Under these conditions, the minimum window thickness (in) $L = L_A + L_B$ required is most nearly:

(A) 0.136
(B) 0.204
(C) 1.6
(D) 2.5



02-04*. The wall of an industrial oven has a thermal conductivity of 0.03 (Btu/h)/(ft·°F). The air inside the oven is at 570°F, and the corresponding convection coefficient is 5.3 (Btu/h)/(ft^2·°F). Additionally, the inner side absorbs a radiant flux of 31.7 (Btu/h)/ft^2 from hot objects inside the oven. The room air outside the oven is 77°F, and the corresponding convection coefficient is 1.8 (Btu/h)/(ft^2·°F). Under these conditions, the required wall thickness (in) so the outer wall surface temperature is 104°F is most nearly:

(A) 0.3

(B) 3.4

(C) 6.9

(D) 9.2

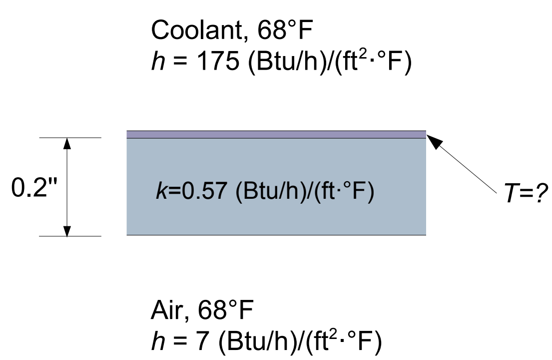
02-05*. A thin, highly-conducting metal heater generates heat at a rate of 9,500 (Btu/h)/ft^2. The top of the heater is bathed by a coolant fluid at 68°F with a convection coefficient of 175 (Btu/h)/(ft^2·°F). The bottom of the heater is in contact with a 0.2-inch thick layer of a material with a conductivity of 0.57 (Btu/h)/(ft·°F), and the bottom of this layer is exposed to air at 68°F with a convection coefficient of 7 (Btu/h)/(ft^2·°F). Under these conditions, the temperature (°F) of the heater is most nearly:

(A) 120

(B) 140

(C) 160

(D) 180



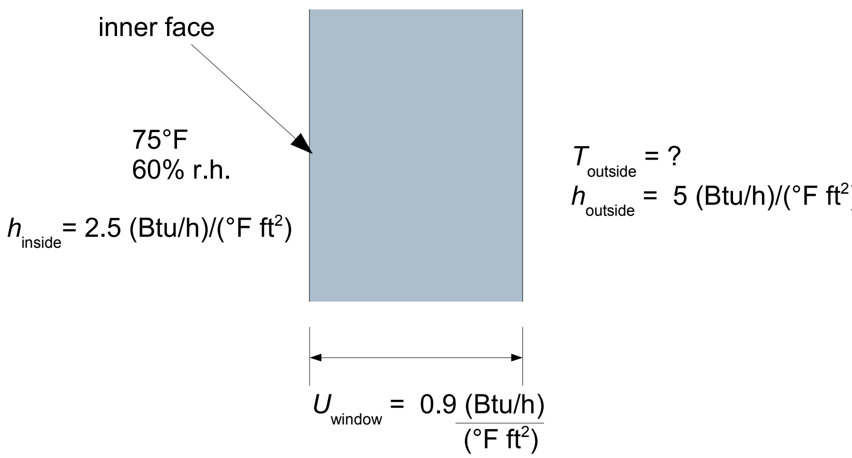
02-06*. An indoor swimming pool room has a 6 ft × 4 ft single-pane, outside wall window with a U-factor of 1.11 (Btu/h)/(°F ft^2). The indoor air film coefficient is 2.5 (Btu/h)/(°F ft^2), and the convection coefficient for the side exposed to outdoor air is 5 (Btu/h)/(°F ft^2). The indoor air is maintained at 75°F with a relative humidity of 60%. Under these conditions, the winter outdoor air temperature (°F) below which condensation on the inner face of the window is expected to form is most nearly:

(A) 0

(B) 8

(C) 19

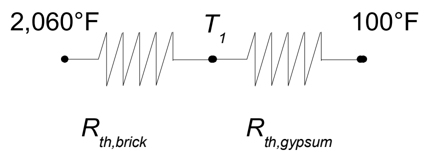
(D) 30



Solutions:

02-01.

a) Draw the equivalent circuit,



We can write the heat transfer rate in terms of the overall temperature difference:

$$q_x = \frac{(2{,}060 - 100)\,°\mathrm{F}}{R_{th,\,total}}$$

and also in terms of either one of the layers:

$$q_x = \frac{(T_1 - 100)\,°\mathrm{F}}{R_{th,\,gypsum}}$$

equating these two expressions, and solving for T_1 we get:

$$\begin{aligned}
T_1 &= 100\,°\mathrm{F} + \frac{R_{th,\,gypsum}}{R_{th,\,total}}(2{,}060 - 100)\,°\mathrm{F} \\
&= 100\,°\mathrm{F} + \left[\frac{\dfrac{0.25\,\mathrm{in}}{0.09\,\dfrac{\mathrm{Btu/h}}{\mathrm{ft}\,°\mathrm{F}}\,A}}{\dfrac{0.25\,\mathrm{in}}{0.09\,\dfrac{\mathrm{Btu/h}}{\mathrm{ft}\,°\mathrm{F}}\,A} + \dfrac{6\,\mathrm{in}}{0.25\,\dfrac{\mathrm{Btu/h}}{\mathrm{ft}\,°\mathrm{F}}\,A}}\right] \times 1{,}960\,°\mathrm{F} \\
&= 303.3\,°\mathrm{F}
\end{aligned}$$

b) We can use the temperature change across the gypsum board to calculate the heat transfer rate:

$$\begin{aligned}
q_x &= \frac{(T_1 - 100)\,°\mathrm{F}}{R_{th,\,gypsum}} \\
&= \frac{(303.3 - 100)\,°\mathrm{F}}{\left(\dfrac{0.25\,\mathrm{in} \times \left|\dfrac{1\,\mathrm{ft}}{12\,\mathrm{in}}\right|}{0.09\,\dfrac{\mathrm{Btu/h}}{\mathrm{ft}\,°\mathrm{F}} \times 6\,\mathrm{ft} \times 6\,\mathrm{ft}}\right)} = 31{,}617\,\mathrm{Btu/h}
\end{aligned}$$

and the heat flux (heat transfer per unit area):

$$q''=\left(\frac{31{,}617\ \mathrm{Btu/h}}{6\ \mathrm{ft}\times 6\ \mathrm{ft}}\right)=878.25\frac{\mathrm{Btu/h}}{\mathrm{ft}^2}$$

c) To find the R-value, use the definition, Equation 2-9:

$$\text{R-value} = R_{th}A = \frac{(0.25/12)\,\mathrm{ft}}{0.09\dfrac{\mathrm{Btu/h}}{\mathrm{ft\,°F}}}+\frac{(6/12)\,\mathrm{ft}}{0.25\dfrac{\mathrm{Btu/h}}{\mathrm{ft\,°F}}} = 2.23\frac{\mathrm{ft^2\,°F}}{\mathrm{Btu/h}}$$

and the U-value is the inverse of the R-value (see Equation 2-11)

$$U = \frac{1}{\text{R-value}} = 0.448\frac{\mathrm{Btu/h}}{\mathrm{ft^2\,°F}}$$

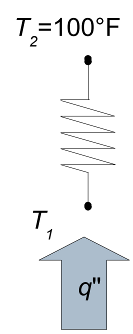
d) Use Equation 1-3

$$T_\infty=T_s-\frac{q''}{h} = 100\,°\mathrm{F}-\frac{878.25\dfrac{\mathrm{Btu/h}}{\mathrm{ft}^2}}{35\dfrac{\mathrm{Btu/h}}{\mathrm{ft^2\,°F}}} \approx 75\,°\mathrm{F}$$

e) Use Equation 1-3

$$h = \frac{q''}{(T_\infty-T_s)} = \frac{878.25\dfrac{\mathrm{Btu/h}}{\mathrm{ft}^2}}{(2{,}150-2060)\,°\mathrm{F}} = 9.76\frac{\mathrm{Btu/h}}{\mathrm{ft^2\,°F}}$$

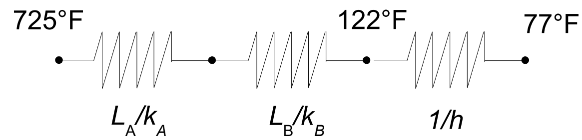
02-02. The heater is thin (small thickness, L) and highly conducting (large conductivity k) so it offers negligible thermal resistance, thus negligible temperature gradients; this means it is isothermal. Furthermore, the heat generated by the heater only flows upward because of the insulator at the bottom. So, the equivalent circuit is as shown in the sketch.



$$q''=\frac{(T_1-100)\,°\mathrm{F}}{L/k} \;\Rightarrow\; T_1=100\,°\mathrm{F}+q''\frac{L}{k}$$

$$T_1 = 100\,°\mathrm{F}+650\frac{(\mathrm{Btu/h})}{\mathrm{ft}^2}\times\frac{(0.25/12)\,\mathrm{ft}}{0.05\dfrac{\mathrm{Btu/h}}{\mathrm{ft\,°F}}} \approx 371\,°\mathrm{F}$$

02-03*. The correct answer is (D). Since we know the temperature at the outer face of the door, we can perform a surface energy balance at this location. The heat entering from the left must equal the heat exiting towards the right:



$$\frac{(725-122)\,°\mathrm{F}}{\dfrac{L_A}{k_A}+\dfrac{L_B}{k_B}} = \frac{(122-77)\,°\mathrm{F}}{\dfrac{1}{h}}$$

Note that there is no convective resistance on the inside (because the convection coefficient there is very large) so the temperature at the inside surface is taken to be identical as the surface of the air inside the oven. Now, insert $L_B = 0.5\,L_A$ and solve for L_A:

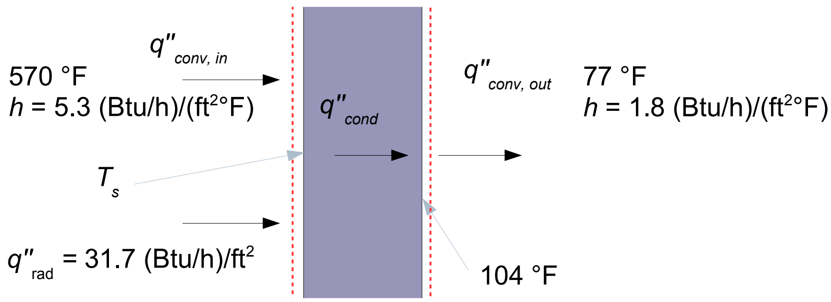
$$L_A = \frac{\dfrac{1}{h}\times\dfrac{(725-122)}{(122-77)}}{\left(\dfrac{1}{k_A}+\dfrac{0.5}{k_B}\right)} = \frac{\dfrac{1}{4.4}\dfrac{(\mathrm{ft}^2\,°\mathrm{F})}{(\mathrm{Btu/h})}\times 13.4}{\left[\dfrac{1}{0.087}+\dfrac{0.5}{0.046}\right]\dfrac{(\mathrm{ft}\,°\mathrm{F})}{(\mathrm{Btu/h})}} = 0.136\,\mathrm{ft}$$

Therefore, the required thickness of the composite wall is:

$$L_A = (0.136 + 0.5\times 0.136)\,\mathrm{ft} = 0.204\,\mathrm{ft} = 2.448\,\mathrm{in}$$

02-04*. The correct answer is (B)

Perform an energy balance on the wall. Entering from the left, we have the radiant and the convection heat transfer rates. This sum must equal the heat transfer rate that exits from the right by convection. The control volume for the energy balance is shown in the red dashed lines:



$$q"_{\text{conv, in}} + q"_{\text{rad}} = q"_{\text{conv, out}}$$

$$31.7\frac{\text{Btu/h}}{\text{ft}^2} + 5.3\frac{\text{Btu/h}}{\text{ft}^2\,°\text{F}}(570 - T_s)°\text{F} = 1.8\frac{\text{Btu/h}}{\text{ft}^2\,°\text{F}}(104-77)°\text{F}$$

$$5.3\frac{\text{Btu/h}}{\text{ft}^2\,°\text{F}}(570 - T_s)°\text{F} = 16.9\frac{\text{Btu/h}}{\text{ft}^2}$$

$$\Rightarrow\ T_s = 566.8\,°\text{F}$$

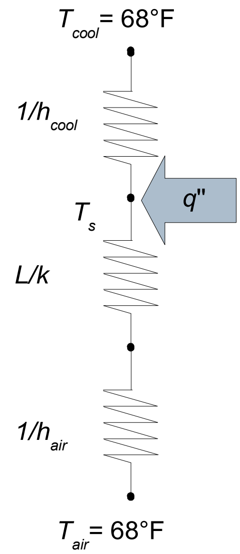
Now that we have the temperature of the inner face, we can solve for the wall thickness L from:

$$q"_{\text{cond}} = q"_{\text{conv, out}}$$

$$\frac{(566.8-104)°\text{F}}{L/k} = 1.8\frac{\text{Btu/h}}{\text{ft}^2\,°\text{F}}(104-77)°\text{F} = 48.6\frac{\text{Btu/h}}{\text{ft}^2}$$

$$\Rightarrow\ L = \frac{462.8\,°\text{F}}{\left(\dfrac{48.6\dfrac{\text{Btu/h}}{\text{ft}^2}}{0.03\dfrac{\text{Btu/h}}{\text{ft}\,°\text{F}}}\right)} = 0.285\,\text{ft} = 3.43\,\text{in}$$

02-05*. The correct answer is (A). The heater is thin (small thickness, *L*) and highly conducting (large conductivity *k*) so it offers negligible thermal resistance, thus negligible temperature gradients; this means it is isothermal. Part of the heat generated by the heater flows upward (towards the coolant on top) and the rest flows downward. The equivalent circuit is as shown in the sketch.



$$9{,}500\,\frac{\text{Btu/h}}{\text{ft}^2} = q"_{up} + q"_{down}$$

$$9{,}500\,\frac{\text{Btu/h}}{\text{ft}^2} = \frac{\left(T_s - T_{cool}\right)}{1/h_{cool}} + \frac{\left(T_s - T_{air}\right)}{L/k + 1/h_{air}}$$

$$9{,}500\,\frac{\text{Btu/h}}{\text{ft}^2} = \frac{\left(T_s - 68\,°\text{F}\right)}{0.0057\dfrac{\left(\text{ft}^2\,°\text{F}\right)}{\text{Btu/h}}} + \frac{\left(T_s - 68\,°\text{F}\right)}{0.172\dfrac{\left(\text{ft}^2\,°\text{F}\right)}{\text{Btu/h}}}$$

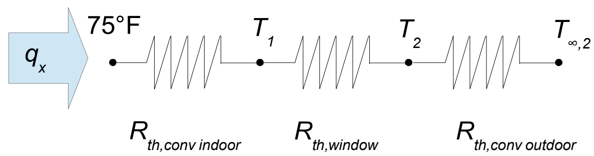
$$9{,}500\,\frac{\text{Btu/h}}{\text{ft}^2} = \left(T_s - 68\,°\text{F}\right) \times 181.25\,\frac{\text{Btu/h}}{\text{ft}^2\,°\text{F}}$$

$$\Rightarrow \quad T_s = 120.4\,°\text{F}$$

02-06*. The correct answer is (C)

We need to determine the dew point of the indoor air, and assign that value to the inner face of the window. Then, we need to calculate the outdoor air temperature.

We have three resistances in series: the indoor air film convection, the window, and the outdoor air film convection resistance.



The total thermal resistance is the sum of the individual resistances. The convection resistances are:

$$R_{th,\,conv\ outdoor} = \frac{1}{h_{outdoor} A} = \frac{1}{5\dfrac{\text{Btu/h}}{\text{°F ft}^2}\times A} \qquad R_{th,\,conv\ indoor} = \frac{1}{h_{indoor} A} = \frac{1}{2.5\dfrac{\text{Btu/h}}{\text{°F ft}^2}\times A}$$

The conduction resistance can be expressed in terms of the given U-factor:

$$R_{th\ cond} = \frac{1}{U\,A} = \frac{1}{1.11\dfrac{\text{Btu/h}}{\text{°F ft}^2}\times A}$$

So, the total resistance is:

$$R_{total} = \frac{1}{A}\left(\frac{1}{2.5}+\frac{1}{1.11}+\frac{1}{5}\right)\frac{\text{°F ft}^2}{\text{Btu/h}} = \frac{1}{A}\times 1.5\,\frac{\text{°F ft}^2}{\text{Btu/h}}$$

Since the heat flux across the indoor air film must equal the heat flux across the entire assembly:

$$\frac{\left(75\,\text{°F}-T_1\right)}{\dfrac{1}{A}\times 0.4\,\dfrac{\text{°F ft}^2}{\text{Btu/h}}} = \frac{\left(75\,\text{°F}-T_{\infty,2}\right)}{\dfrac{1}{A}\times 1.5\,\dfrac{\text{°F ft}^2}{\text{Btu/h}}}$$

where each side of the above equation is equal to q_x. We can solve for $T_{\infty,2}$ to obtain:

$$T_{\infty,2} = 75\,\text{°F} - \frac{1.5}{0.4}\left(75\,\text{°F}-T_1\right)$$

where $T_1 = 60\,\text{°F}$ is the dew point for the indoor space. Therefore:

$$T_{\infty,2} = 75\,\text{°F} - \frac{1.5}{0.4}\times\left(75-60\right)\text{°F} = 18.75\,\text{°F}$$

