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MECHANICAL ENGINEERING
THERMAL AND FLUID SYSTEMS
STUDY PROBLEMS

HEAT TRANSFER & HEAT EXCHANGERS

How to use this book

The exam specifications in effect since April 2017 state that approximately 6 problems from the "Heat Transfer Principles" topic will be in the morning Principles portion of your exam. Reviewing all the problems in this book will prepare you for all these problems in the morning Principles portion.

Furthermore, Heat Exchangers are listed as one of the relevant examples of Energy/Power Equipment in the afternoon Energy/Power System Applications portion. Reviewing all the problems in this book will prepare you for these problems in the Energy/Power System Applications portion of the exam.

How it works

This study problems book works on what we call the "principle of progressive overload". With this technique you start with very easy problems and smoothly progress towards more complex problems. A good example of progressive overload is the story of the famous wrestler Milo of Croton in ancient Greece. This extraordinarily strong man was allegedly capable of carrying a fully grown bull on his shoulders. He was reported to have achieved this tremendous strength by walking around town with a new born calf on his shoulders every single day. As the calf grew, so did the man's strength.

We recommend you work the problems in this book in the order they are presented. Within each section of the book, the first problems will feel "light", like carrying that baby calf – you might even be tempted to skip them. We strongly urge you to resist this temptation. As you progress, the problems become harder, but the work you've been putting in with all the previous problems will bear fruit. You will be pleasantly surprised at how relatively easy those "hard" problems will seem. You will soon be carrying intellectual bulls on your shoulders! **The problems that are considered "exam-level difficult" are denoted with an asterisk**.

This book is comprised of the following sections:

For the most part, these sections are not independent and build from the previous ones. We recommend you go through them in the order presented, and be sure to review them all. Each section begins with a brief discussion of the relevant concepts and equations. These discussions are laser-focused on the aspects that are relevant to the P.E. exam and do not go into derivations with academic rigor.

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SECTION 01: Introduction

Heat Transfer occurs whenever a temperature difference exists in a medium or between media. We refer to different types of heat transfer processes as modes. **Conduction** refers to the heat transfer that occurs across a (solid or fluid) medium. In contrast, the term **convection** refers to heat transfer that occurs between a surface and a moving fluid when they are at different temperatures. Finally, all surfaces of finite temperature emit energy in the form of electromagnetic waves, a phenomenon we call **thermal radiation**. Hence, in the absence of a participating medium, there is net heat transfer by radiation between two surfaces at different temperatures.

When a furnace wall feels warm to the touch (but contains a controlled fire inside) you are witnessing an example of thermal conduction (heat transfer across the wall due to molecular activity). A room in the summer time gains heat through the walls through conduction.

Consider a wall of thickness L with one side of the wall at a temperature T_1 and the other side at a temperature T_2 , as shown in Figure 1-1. If T_1 and T_2 don't change with time, we have a steady-state. If the wall is much taller and wider than it is thick, then the direction of heat flow will solely be along the x axis of Figure 1-1. Therefore, the heat transfer is one-dimensional.

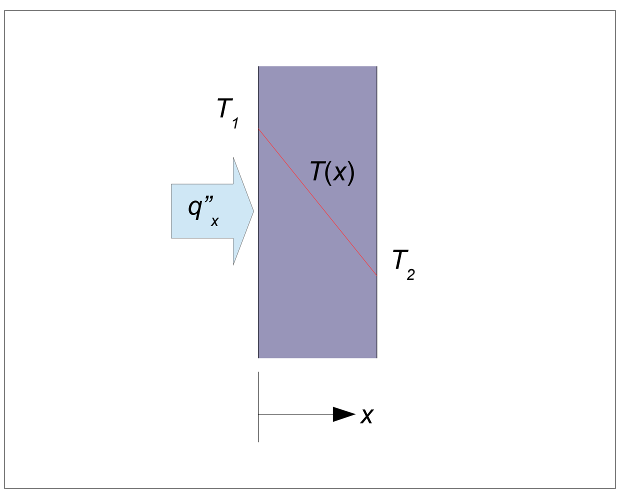


Figure 1-1. One-dimensional, steady conduction through a wall

The rate of heat transfer by conduction is given by **Fourier's law**,

$$q_x" = -k \frac{dT}{dx} \tag{1-1}$$

The heat flux $q_x"$ is the heat transfer rate in the x-direction per unit area perpendicular to the direction of transfer, and it is proportional to the temperature gradient, dT/dx, in this direction. The parameter k is the thermal conductivity and is a characteristic of the wall material. Heat is transferred in the direction of decreasing temperature, hence the negative sign in Equation (1-1). Under the steady-state conditions shown in Figure 1-1, where the temperature distribution is linear, the temperature gradient is:

$$\frac{dT}{dx}=\frac{T_2-T_1}{L}$$

so the heat flux is:

$$q_x" = k\frac{T_1-T_2}{L} = k\frac{\Delta T}{L} \quad (1\text{-}2)$$

Note that Equation (1-2) provides a heat flux, or heat transfer rate per unit area perpendicular to the flow direction. The heat rate by conduction, q_x (Btu/hour or W), through a plane wall of area A is the product of the flux and the area: $q_x = q_x" A$.

In SI units, heat flux is in W/m^2. In US Customary System, heat flux is typically in (Btu/hour)/ft^2 or (Btu/hour)/in^2. Similarly, the thermal conductivity is in W/(m·°C) in SI units, and in (Btu/hour)/(ft·°F) something equivalent such as (Btu·in/hour)/(ft^2·°F). You might encounter W/(m·K) or (Btu/hour)/(ft·°R) that is, with the absolute temperature units in the denominator. This is inconsequential because (Btu/hour)/(ft·°R) and (Btu/hour)/(ft·°F) are completely interchangeable without a need to convert between them. The reason is that thermal conductivity is always multiplied by a temperature difference (as in Equation (1-2)) so a ΔT of, say, 25°F is also a ΔT of 25°R.

For the purposes of the PE exam, we are especially interested in convection heat transfer, when it occurs between a fluid in motion and a bounding surface when the two are at different temperatures. Convection heat transfer may be classified according to the nature of the flow.

Forced convection occurs when the flow is caused by external means, such as by a fan or pump. As an example, consider the use of a fan to provide forced convection air cooling of hot tubes and fin in an automotive radiator. In contrast, for **free (or natural) convection**, the flow is induced by buoyancy forces, which are due to density differences caused by temperature variations in the fluid. An example is the free convection heat transfer that occurs from hot metal parts removed from a heat treatment furnace and placed in a quiescent pool of cool liquid for quenching. The liquid that makes contact with the metal parts experiences an increase in temperature and hence a reduction in density. Since it is now lighter than the surrounding liquid, buoyancy forces induce a vertical motion for which warm liquid ascending from the metal parts is replaced by an inflow of cooler liquid.

There are situations in which **mixed convection** exists, which is when the effects of forced and natural convection are comparable. An important subset of convection problems involve **boiling and condensation**. In these situations, the effects of phase-change and the transfer of latent heat are dominant.

Independent of the nature of the convection heat transfer process, the appropriate rate equation is:

$$q'' = h\left(T_s - T_\infty\right) \tag{1-3}$$

where q'', the convective heat flux is proportional to the difference between the surface and fluid temperatures, T_s and T_∞ respectively. This expression is known as Newton's law of cooling, and the parameter h is the convection heat transfer coefficient, or film coefficient. This coefficient depends on conditions in the boundary layer, which are influenced by surface geometry, the nature of the fluid motion, and an assortment of fluid thermodynamic and transport properties.

For a fixed heat flux q'', a small h means a high temperature difference between the surface and the bathing fluid. Conversely, a high h means a low temperature difference between the surface and the bathing fluid. You might see in a problem statement a phrase such as: "*a very large heat transfer coefficient*", which means that it is safe to assume that the surface and the bathing fluid are practically at the same temperature. Convection heat transfer analysis generally consists of determining the convection coefficient.

The units of h in SI are typically W/(m^2·°C) and Btu/(hour·ft^2·°F) in the US Customary System. You might encounter W/(m^2·K) or (Btu/hour)/(ft^2·°R) that is, with the absolute temperature units in the denominator. This is inconsequential because (Btu/hour)/(ft^2·°R) and (Btu/hour)/(ft^2·°F) are completely interchangeable without a need to convert between them. The reason is that the heat transfer coefficient is always multiplied by a temperature difference (as in Equation (1-3)) so a ΔT of, say, 25°F is also a ΔT of 25°R.

Thermal radiation is energy emitted by matter that is at a nonzero absolute temperature. The energy is transported by electromagnetic waves (or alternatively, photons). While the transfer of energy by conduction or convection requires the presence of a material medium, radiation does not. The energy from the sun that reaches Earth's atmosphere does so by radiation.

A surface at an absolute temperature T_s will emit energy at a certain rate per unit area, known as the emissive power E. A theoretically maximum emissive power E_b is given by the **Stefan-Boltzmann law**:

$$E_b = \sigma T_s^4 \tag{1-5}$$

where $\sigma = 5.67\times10^{-8}\,\mathrm{W/(m^2\cdot K^4)} = 1.714\times10^{-9}\,(\mathrm{Btu/h})/(\mathrm{ft^2\,{}^\circ R^4})$ is the Stefan-Boltzmann constant. A surface for which the emissive power is given by equation (1-5) is known as a **blackbody**, or an **ideal radiator**.

The emissive power of a real surface is less than that of the ideal radiator and is given by:

$$E = \epsilon\,\sigma T_s^4 \tag{1-6}$$

where ϵ is the emissivity, a property that depends on the surface finish and material. The emissivity is always less than one, so it provides an efficiency of thermal radiation emission with respect to that of a blackbody.

Radiation may also be incident on, (or received by) a surface. Incident radiation on a surface may come from the sun, a flame, a fireball, or other surfaces to which the surface is exposed to. The rate at which all such radiation is incident on a unit area of the surface as the irradiation G.

The rate at which irradiation is absorbed may be evaluated from knowledge of a surface radiative property termed the absorptivity α as follows:

$$G_{\text{abs}} = \alpha G \tag{1-7}$$

where $0 \leqslant \alpha \leqslant 1$. A surface for which $\alpha = \epsilon$ is called a **gray surface**. In addition to being absorbed, irradiation may be reflected (by opaque surfaces) and transmitted (by semitransparent surfaces). The value of α depends on the nature of the irradiation, as well as on the surface itself. For example, the absorptivity of a surface to solar radiation may differ from its absorptivity to radiation emitted by the walls of a furnace or by other heat sources.

A common case occurs when a small surface at absolute temperature T_s is completely surrounded by a much bigger isothermal surface (which we call "the surroundings") at T_∞. It can be shown that the incident radiation on the small surface is $G=\sigma T_{\infty}^{4}$. If the emissive power for the small surface is $E=\epsilon E_{b}(T_{s})$, then the net thermal radiation from the surface to the surroundings is $q''_{\text{rad}}=E-\alpha G$. If we assume the small surface is gray, then:

$$q''_{\text{rad}}=\epsilon \sigma \left(T_{s}^{4}-T_{\infty}^{4}\right) \tag{1-8}$$

It is convenient sometimes to linearize Equation (1-6) as:

$$q''_{\text{rad}}=h_{\text{rad}}\left(T_{s}-T_{\infty}\right) \tag{1-9}$$

where h_{rad} is the radiation heat transfer coefficient defined as:

$$h_{\text{rad}}=\epsilon \sigma \left(T_{s}+T_{\infty}\right)\left(T_{s}^{2}+T_{\infty}^{2}\right) \tag{1-10}$$

In heat transfer problems, it is common to apply an energy balance on a surface. The control volume surfaces are located on either side of the physical boundary. Such control volumes enclose no mass, so energy can't be stored in these control volumes. An example is shown in Figure 1-2.

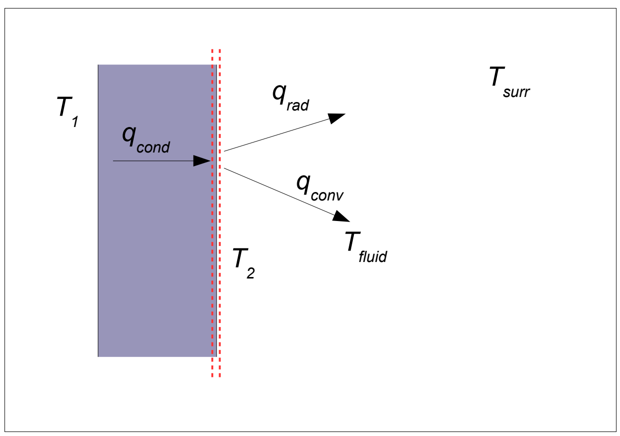


Figure 1-2. Surface energy balance

The figure shows a wall that is subject to a temperature gradient across its thickness. The left side is at a temperature T_{1} and the right side at T_{2}. This gradient drives a conductive heat transfer across the solid wall. The right side is bathed by a fluid of temperature T_{fluid}, which drives a convective heat transfer from the right wall surface to the fluid. Finally, the surroundings are at T_{surr} therefore, a net radiative transfer exists between the right side of the wall and the surroundings. The surface control volume is indicated with dashed red lines. The rate at which energy enters the control volume must equal the rate at which it leaves, so for the example of Figure 1-2, the surface energy balance is:

$$q''_{\text{cond}}=q''_{\text{rad}}+q''_{\text{conv}}$$

PROBLEMS

01-01. The wall of an industrial furnace is built from a material with a thermal conductivity of 1.0 (Btu/h)(ft·°F). Steady-state operating temperatures of 2060 °F and 1610 °F are recorded at the inner and outer surfaces, respectively. What is the rate of heat loss through a wall that is 20 in × 48 in on a side?

01-02. A heat transfer rate of 5 kW is conducted through a section of an insulating material of cross-sectional area 10 m^2 and thickness 2.5 cm. If the inner (hot) surface temperature is 375°C and the thermal conductivity of the material is 0.12 W/(m·K), what is the outer surface temperature?

01-03. A laboratory hot-plate heater with a diameter of 6 inches is left on and it reaches a steady-state temperature of 500°F at the surface. The power delivered from the heating element to the plate is 340 Btu/hour. The average bulk temperature of the air surrounding the plate is approximately 85°F. Assume all heat transfer from the plate occurs by convection only. Calculate the convective coefficient.

01-04*. A freezer compartment consists of a cubical cavity that is 6.5 ft on a side with the bottom side perfectly insulated. The normal operating temperatures of the inner and outer surfaces are 15°F and 95°F, respectively. Under these conditions, the thickness of styrofoam insulation $\left(k = 0.017\,\text{Btu/h/ft/°F}\right)$ that must be applied to the top and side walls to ensure a heat load of 1,700 Btu/h, is most nearly:

(A) 0.17

(B) 1.5

(C) 2

(D) 2.5

01-05*. The wall of an industrial heat treatment oven is 6-in thick and composed of fireclay brick (thermal conductivity of 1 (Btu/h)/(ft·°F)). The inner face is subject to a heat flux of 35 (Btu/h)/ft^2. The opposite face is exposed to air at 85°F with a convection coefficient of 3.5 (Btu/h)/(ft^2·°F). Under steady state operation, the temperature (°F) of the wall surface exposed to air is most nearly:

(A) 90

(B) 95

(C) 100

(D) 105

01-06*. The wall of an industrial heat treatment oven is 8-in thick and composed of fireclay brick (thermal conductivity of 0.93 (Btu/h)/(ft·°F)). The inner face is subject to a heat flux of 25 (Btu/h)/ft^2. The opposite face is exposed to air at 75°F with a convection coefficient of 3.5 (Btu/h)/(ft^2·°F). Under steady state operation, the temperature (°F) of the inner wall surface air is most nearly:

(A) 82

(B) 96

(C) 100

(D) 297

01-07*. A wall has inner and outer surface temperatures of 15°C and 6°C, respectively. The interior and exterior air temperatures are 20°C and 5°C, respectively. The outer convection heat transfer coefficient is 20 W/(m^2K). The inner convection heat transfer coefficient (W/(m^2K)) is most nearly:

(A) 4

(B) 10

(C) 50

(D) 100

01-08*. An electric resistance heater is embedded in a 3 feet long cylinder of 1¼ inch diameter. When water with a temperature of 77°F and velocity of 3.5 ft/s flows crosswise over the cylinder, the power consumed by the heater to maintain the surface at a uniform temperature of 195°F is 25 kW. Under these conditions, the heat transfer coefficient ((Btu/h)/(ft^2·°F)) for the water bathed surface of the cylinder is most nearly:

(A) 190

(B) 370

(C) 520

(D) 740

01-09*. An automotive transfer case transfers power from the transmission to the rear axles with an efficiency of 90%. The transfer case can be modeled as a cube of 1 ft length, cooled by 85°F air and a convective coefficient of 35 (Btu/h)/(ft^2·°F) on all sides. When the power from the transmission is 150 hp, the average surface temperature (°F) of the transfer case is most nearly:

(A) 85

(B) 267

(C) 295

(D) 1,200

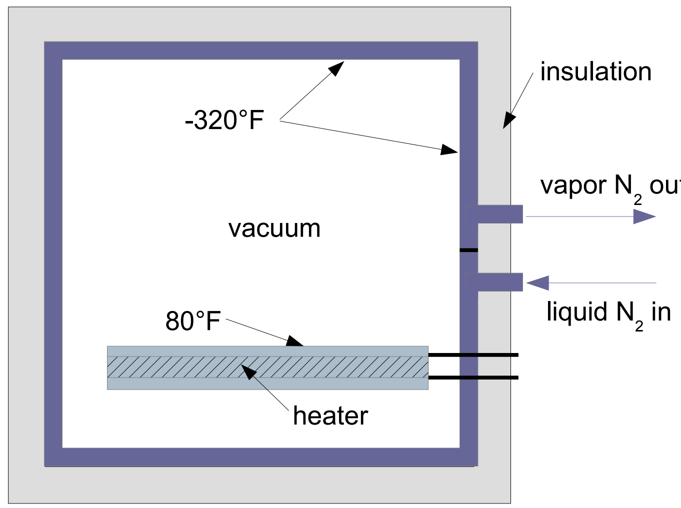
01-10*. The manufacturing process of thin films on microcircuits uses a perfectly insulated vacuum chamber whose walls are kept at -320°F by a liquid nitrogen bath. An electric resistance heater is embedded inside a 1.5 feet long cylinder of 1½ inch diameter placed inside the vacuum chamber. The surface of the cylinder has an emissivity of 0.25 and is maintained at 80°F by the heater. The nitrogen enters the chamber bath as a saturated liquid and leaves as a saturated vapor. Neglecting any heat transfer from the ends of the cylinder, the required flow rate of nitrogen (pounds-mass per hour) is most nearly (the latent heat of vaporization for N_2 is 53.74 Btu per pound):

(A) 0.05

(B) 0.1

(C) 0.2

(D) 0.4



01-11*. A laboratory hot-plate heater with a diameter of 6 inches and an emissivity of 0.87 is left on and it reaches a steady-state temperature of 500°F at the surface. The average bulk temperature of the air surrounding the plate is approximately 100°F, the convection heat transfer coefficient is 2.5 (Btu/h)/(ft^2 °F) and the walls and ceiling of the lab are at 65°F. Under these conditions, the total heat transfer rate (Btu per hour) from the hot plate is most nearly:

(A) 212

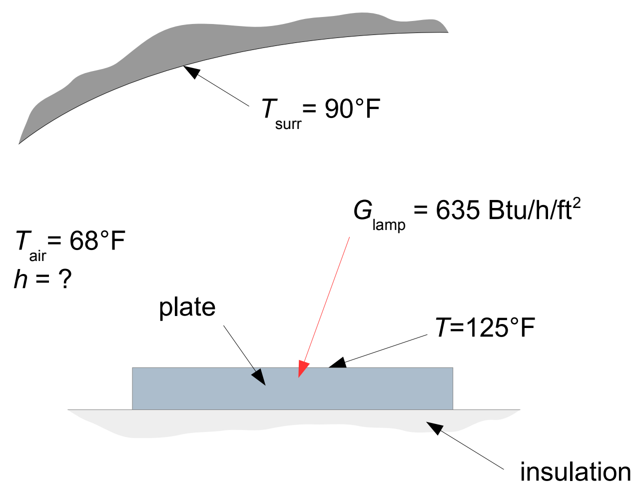
(B) 392

(C) 1,994

(D) 56,384

01-12*. An infrared lamp providing a uniform irradiation of 635 (Btu/h)/ft^2 is utilized to cure the coating on a plate. The plate absorbs 80% of the irradiation from the lamp. It is also exposed to an airflow of 68°F and large surroundings at 90°F. Assume a radiative heat transfer coefficient of 0.63 (Btu/h)/(ft^2 °F) for heat exchange with the surroundings. The process requires a surface temperature of 125°F. Under these conditions, the convective coefficient in (Btu/h)/(ft^2 °F) is most nearly:

(A) 0.63
(B) 4.2
(C) 8.5
(D) 10.8



SECTION 02: 1D Steady State Conduction – The Plane Wall

In this section we continue the discussion on heat conduction and introduce some advanced concepts, but still staying within the context of one-dimensional, steady state problems.

<u>The Plane Wall</u>

Consider the wall of thickness L and thermal conductivity k Figure 2-1. This wall separates a hot fluid at $T_{\infty,1}$ from a cold fluid at $T_{\infty,2}$. It can be shown that the temperature variation *within the wall* is linear from a value T_1 (for the surface in contact with the hot fluid) to a value T_2 (for the surface in contact with the cold fluid).

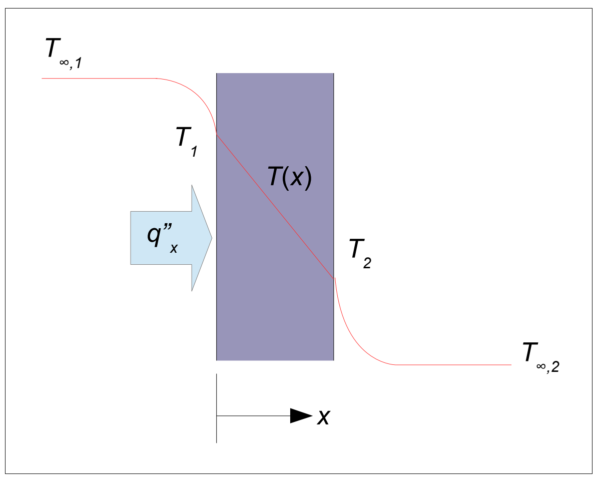


Figure 2-1. Plane wall with convective heat transfer on both sides

From Fourier's law, in the situation of Figure 2-1, the heat transfer rate is

$$q_x = \frac{k\,A}{L}\left(T_1 - T_2\right) \tag{2-1}$$

where A is the wall area perpendicular to the direction of the flow of heat.

Equation (2-1) indicates that – for a plane wall – the heat transfer rate is a constant, independent of x. The temperature difference $\Delta T = \left(T_1 - T_2\right)$ can be interpreted as a "driving potential" that triggers the transfer of heat. Whenever $\Delta T \neq 0$ there will be heat transfer. For a fixed value of $\Delta T \neq 0$, the magnitude of the heat transfer can be varied with the magnitude of the parameter $L/(k\,A)$, which is known as the **thermal resistance for conduction**, $R_{t,\text{cond}}$:

$$R_{th,\,\mathrm{cond}} = \frac{L}{k\,A} \tag{2-2}$$

So that Equation (2-1) can be written as $q_x = \Delta T / R_{t,\,\mathrm{cond}}$. For a fixed temperature change, the higher the thermal resistance, the lower the heat transfer rate and vice-versa. Note that the thermal resistance decreases with increasing conductivity and with decreasing thickness. Therefore, thin layers of highly conducting materials (for example thin walled metal pipes) experience very small temperature gradients, for

The concept of thermal resistance can be developed for convection as well. Consider the right side of the wall in Figure 2-1. The heat flux can be expressed as $q_x = (T_2 - T_{\infty,2}) / R_{t,\,\mathrm{conv}}$ where the **thermal resistance for convection**, is defined as:

$$R_{th,\,\mathrm{conv}} = \frac{1}{h\,A} \tag{2-3}$$

A t**hermal resistance for radiation** is defined the same way as in Equation (2-3) but the coefficient h is the radiation coefficient we defined in Equation (1-10).

In Figure 2-1 we see that the heat is being transferred from the hot fluid to the cold fluid, because there is a non-zero, overall temperature difference, $T_{\infty,1} - T_{\infty,2}$. In order to do this, the heat has to "make it through" three resistances: a convective resistance on the left side, a conductive resistance, and another convective resistance at the right side. This is analogous to the flow of current due to a voltage potential across resistors. The thermal resistances in the example of Figure 2-1 are in series, and we can develop the analogous "thermal circuit" of Figure 2-2:

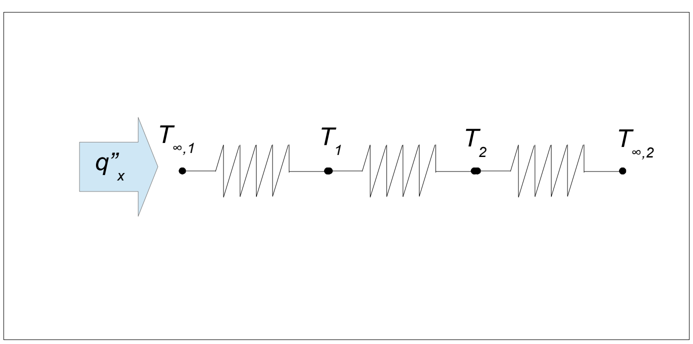


Figure 2-2. "Thermal Circuit" for Plane wall with convective heat transfer on both sides of Figure 2-1

So, just like in the analysis of circuits, we have multiple expressions for the heat transfer rate (current intensity):

$$q_x = \frac{T_{\infty,1}-T_1}{1/(h_1 A)} = \frac{T_1-T_2}{L/(k A)} = \frac{T_2-T_{\infty,2}}{1/(h_2 A)} \tag{2-4}$$

and, like in the analysis of electrical circuits, it can be shown that the three resistance in series can be added when defining a total resistance R_{th} , so that:

$$q_x = \frac{T_{\infty,1}-T_{\infty,2}}{R_{th}} \tag{2-5}$$

where the total resistance for the example of Figure 2-1 would be defined as:

$$R_{th} = \frac{1}{h_1 A} + \frac{L}{k A} + \frac{1}{h_2 A} \tag{2-6}$$

We may also use equivalent thermal circuits for more complex systems, such as **composite walls**. Such walls may involve any number of series and parallel thermal resistances due to layers of different materials. Consider the series composite wall of Figure 2-3:

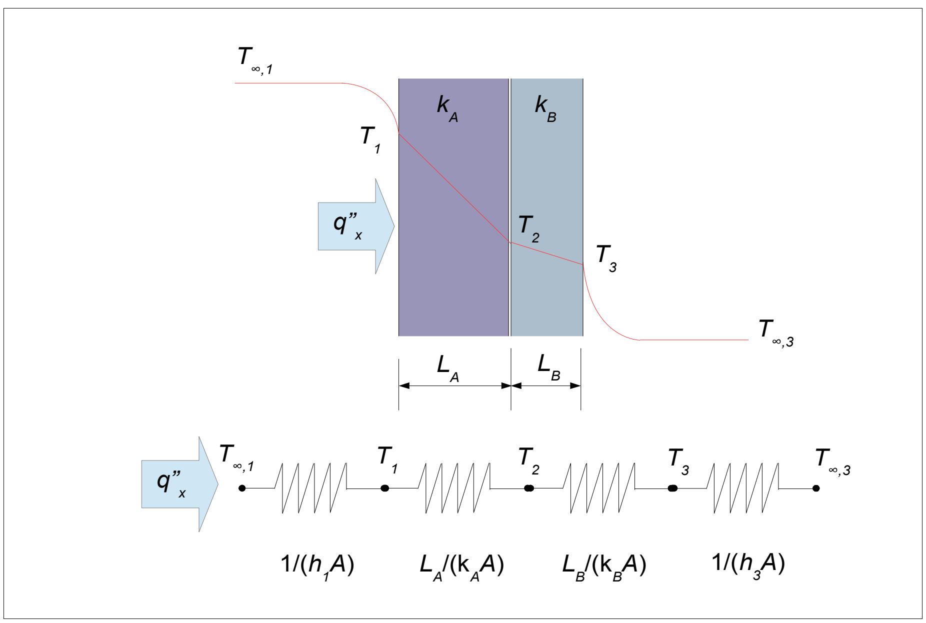


Figure 2-3. Series composite wall, and equivalent resistances

We can write the heat transfer rate in terms of the overall temperature difference:

$$q_x = \frac{T_{\infty,1}-T_{\infty,3}}{1/(h_1 A)+L_A/(k_A A)+L_B/(k_B A)+1/(h_3 A)} \tag{2-7}$$

where the group of terms in the denominator comprise the total resistance. We can also express the heat transfer rate in terms of the individual elements,

$$q_x = \frac{T_{\infty,1} - T_q}{1/(h_1 A)} = \frac{T_1 - T_2}{L_A/(k_A A)} = \frac{T_2 - T_3}{L_B/(k_B A)} = \frac{T_3 - T_{\infty,3}}{1/(h_3 A)}$$

or in terms of partial components of the entire wall, for example,

$$q_x = \frac{T_{\infty,1} - T_2}{1/(h_1 A) + L_A/(k_A A)}$$

which can be used to determine temperatures at the interfaces between components (like T_2, for example).

Resistances can be also arranged in parallel, such as in the situation shown in Figure 2-4. In this case, the radiative and convective resistances are in parallel.

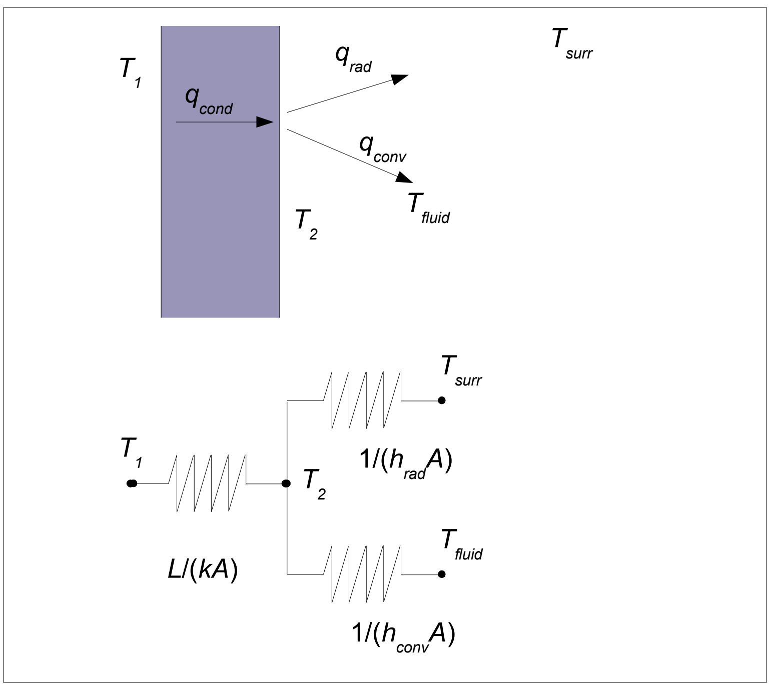


Figure 2-4. Resistances in parallel

If the fluid (for convection) and the surroundings (for radiation) are at the same temperature, $T_{\text{fluid}} = T_{\text{surr}} = T_\infty$ then the circuit analogy can be used to develop an expression for the equivalent resistance from convection and radiation. For the example of Figure 2-4 the resistances in parallel are combined as:

$$\frac{1}{R_{th\,\text{conv,rad}}}=\left(\frac{1}{h_{\text{conv}}A}\right)^{-1}+\left(\frac{1}{h_{\text{rad}}A}\right)^{-1}$$
$$\Rightarrow R_{th,\,\text{conv,rad}}=\frac{1}{A\left(h_{\text{conv}}+h_{\text{rad}}\right)}$$

The heat flux expressed in terms of the overall temperature difference would be:

$$q_x=\frac{T_1-T_\infty}{R_{th,\,\text{total}}}=\frac{T_1-T_\infty}{R_{th,\,\text{cond}}+R_{th,\,\text{conv \& rad}}}=\frac{T_1-T_\infty}{R_{th,\,\text{cond}}+R_{th,\,\text{conv \& rad}}}=\frac{T_1-T_\infty}{\frac{1}{A}\left(\frac{L}{k}+\frac{1}{h_{\text{conv}}+h_{\text{rad}}}\right)}$$

Up to now, we have assumed that there is one unique value of temperature at the interface between two layers in a composite wall. For example, the right side of layer "A" and the left side of layer "B" in Figure 2-3 are both at T_2 . In reality, the surfaces in contact have some roughness associated with it and there are small air gaps and pockets. These air gaps represent another resistance so the mating sides in "real-world" interfaces will be at different temperatures. We model this as a discontinuity in the temperature profile, as shown in Figure 2-5:

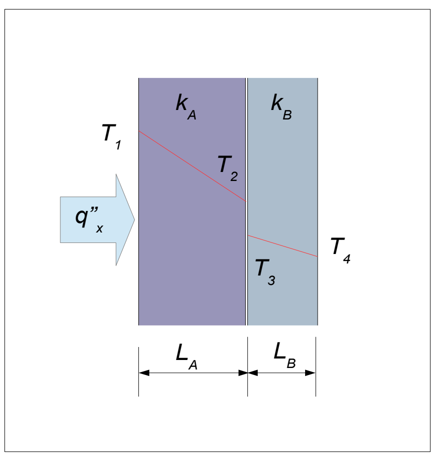


Figure 2-5. The effect of non-negligible contact resistance.

In the figure we see that the temperature drops from T_2 to T_3 because of the contact resistance. The contact resistance is thus defined as:

$$R_{th,\,\text{contact}}=\frac{\left(T_2-T_3\right)}{q"_x} \tag{2-8}$$

Values of contact resistance (per unit area normal to heat flow) are listed in the literature as a function of the materials of the layers, the contact pressure, and the interstitial fluid between layers.

Sometimes (especially in the construction industry) you will encounter the term "**R-value**". The R-value of a layer of material is the thermal resistance per unit area. Typical units are (°F ft^2)/(Btu/h). The R-value is not the same as the thermal resistance we have been discussing so far. They are however, related by:

$$\text{R-value} = R_{th} A = \frac{\Delta T}{q_x''} \tag{2-9}$$

Given the definition of R-value, and that of thermal resistance for convection, it can be seen that the R-value for a layer of fluid is the inverse of the convection coefficient. Also for flat walls in which the area remains constant, the heat flux q_x'' is also constant, so R-values in series can be added to find equivalent R-values.

With composite systems, it is often convenient to work with an **overall heat transfer coefficient** U , also known as the **U-factor** which is defined by:

$$q_x = UA\Delta T \tag{2-10}$$

where ΔT is the overall temperature difference and, for the example of Figure 2-3:

$$U = \frac{1}{R_{th} A} = \frac{1}{1/h_1 + L_A/k_A + L_B/k_B + 1/h_3} \tag{2-11}$$

Note that the U-factor is the inverse of the R-value.

PROBLEMS

02-01. The wall of an industrial furnace is built from 6-in thick fireclay brick and it is covered with a 1/4-inch gypsum board layer. Steady-state operating temperatures of 2,060 °F and 100 °F are recorded at the inner and outer surfaces, as shown in the sketch. Neglecting any thermal radiation, calculate:

a) The temperature (°F) at the brick-gypsum interface.

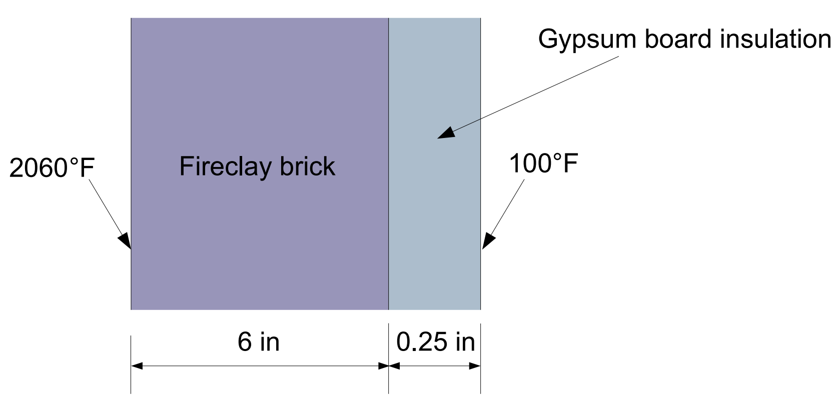
b) The heat flux (Btu/h)/(ft^2) and the heat transfer rate (Btu/h) across the wall, if its dimensions are 6 ft × 6 ft.

c) The R-value and U-factor for the composite wall.

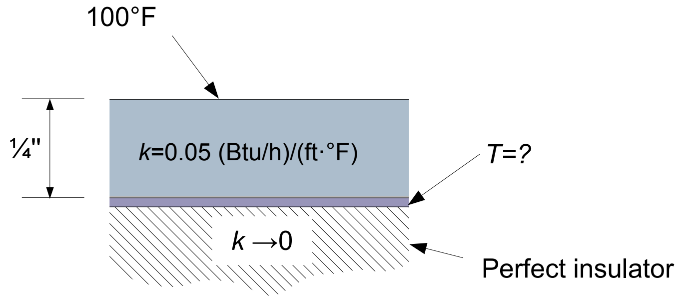
d) The air room temperature on the gypsum board side if the convective coefficient at the exposed side of the gypsum board insulation is 35 (Btu/h)/(ft^2·°F).

e) The convective coefficient on the exposed side of the fireclay brick wall, if the average temperature of the gases inside the furnace is 2150°F.

	Thermal Conductivity (Btu/h)/(ft·°F)
Fireclay brick	0.25
Gypsum board	0.09

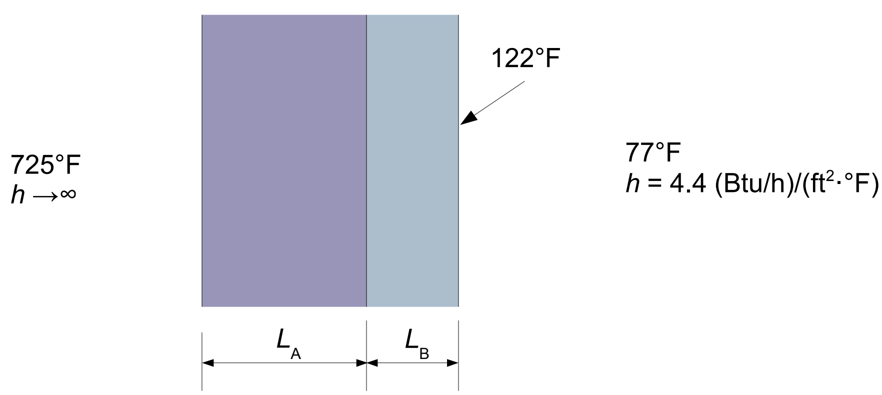


02-02. A thin, highly-conducting metal heater generates heat at a rate of 650 (Btu/h)/ft^2. The heater is placed on a non-conducting substrate (a perfect insulator). There is a 1/4-inch thick layer of a material with a conductivity of 0.05 (Btu/h)/(ft·°F) on top of the heater. The top surface of this layer is kept at 100°F. Find the temperature of the heater.



02-03*. A self-cleaning residential convection oven uses a composite window. The window consists of two high-temperature plastics (A and B) of thicknesses $L_A = 2\,L_B$ and thermal conductivities k_A of 0.087 (Btu/h)/(ft·°F) and k_B of 0.046 (Btu/h)/(ft·°F). During steady state operation at the highest setting, the air inside the oven is 725°F, while the room air temperature is 77°F. The inside convection coefficient is infinitely large and the outside convection coefficient is 4.4 (Btu/h)/(ft^2·°F). To avoid a skin contact burn hazard, the outer surface window shall not exceed 122°F. Under these conditions, the minimum window thickness (in) $L = L_A + L_B$ required is most nearly:

(A) 0.136
(B) 0.204
(C) 1.6
(D) 2.5



02-04*. The wall of an industrial oven has a thermal conductivity of 0.03 (Btu/h)/(ft·°F). The air inside the oven is at 570°F, and the corresponding convection coefficient is 5.3 (Btu/h)/(ft^2·°F). Additionally, the inner side absorbs a radiant flux of 31.7 (Btu/h)/ft^2 from hot objects inside the oven. The room air outside the oven is 77°F, and the corresponding convection coefficient is 1.8 (Btu/h)/(ft^2·°F). Under these conditions, the required wall thickness (in) so the outer wall surface temperature is 104°F is most nearly:

(A) 0.3

(B) 3.4

(C) 6.9

(D) 9.2

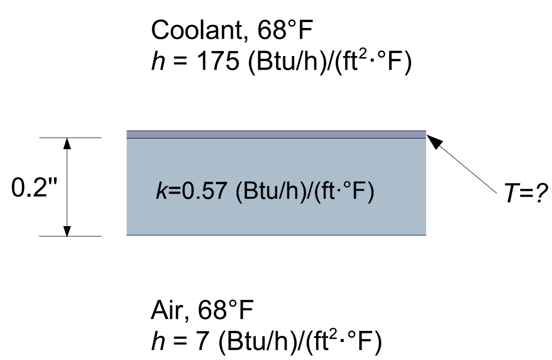
02-05*. A thin, highly-conducting metal heater generates heat at a rate of 9,500 (Btu/h)/ft^2. The top of the heater is bathed by a coolant fluid at 68°F with a convection coefficient of 175 (Btu/h)/(ft^2·°F). The bottom of the heater is in contact with a 0.2-inch thick layer of a material with a conductivity of 0.57 (Btu/h)/(ft·°F), and the bottom of this layer is exposed to air at 68°F with a convection coefficient of 7 (Btu/h)/(ft^2·°F). Under these conditions, the temperature (°F) of the heater is most nearly:

(A) 120

(B) 140

(C) 160

(D) 180



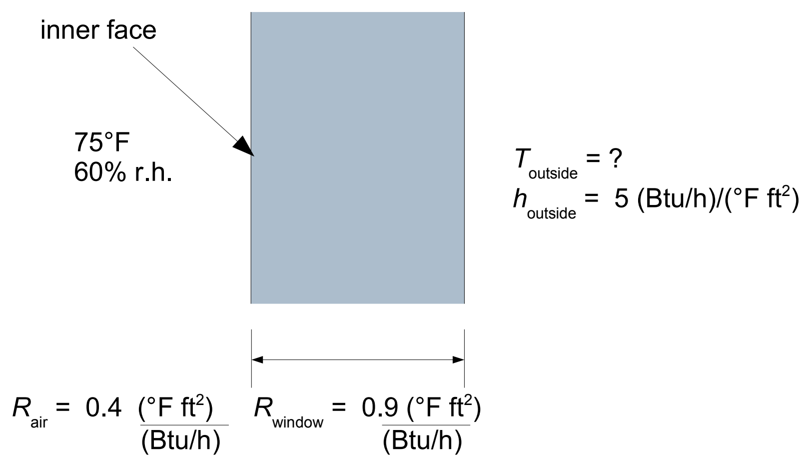
02-06*. An indoor swimming pool room has a 6 ft × 4 ft single-pane, outside wall window with an effective R-value of 0.9 (°F ft^2)/(Btu/h). The R-value for the indoor air film is 0.4 (°F ft^2)/(Btu/h), and the convection coefficient for the side exposed to outdoor air is 5 (Btu/h)/(°F ft^2). The indoor air is maintained at 75°F with a relative humidity of 60%. Under these conditions, the winter outdoor air temperature (°F) below which condensation on the inner face of the window is expected to form is most nearly:

(A) 0

(B) 8

(C) 19

(D) 30



02-07*. A 2m by 2.5m composite wall is composed of two materials, A and B. Material A has a thickness of 10 mm, a thermal conductivity of 0.1 W/(m·K), and it is bathed by a fluid at 200°C with a convection coefficient of 10 W/(m^2·K). Material B has a thickness of 20 mm, a thermal conductivity of 0.04 W/(m·K), and it is bathed by a fluid at 40°C with a convection coefficient of 20 W/(m^2·K). The contact resistance at the interface between the two materials is 0.30 m^2·K/W. Under these conditions, the temperature drop (°C) across the interface between the two materials is most nearly:

(A) 0

(B) 21

(C) 42

(D) 160

SECTION 03: 1D Steady Conduction – Radial Systems & Fins

Radials Systems

Cylindrical and spherical systems often experience temperature gradients in the radial direction only and may therefore be treated as one-dimensional. Consider a **cylindrical shell** of length L , inner radius r_1 and outer radius r_2 shown in Figure 3-1. The inner face is at a temperature T_1 , while it is bathed by a fluid at a temperature $T_{\infty,1}$ (this could be the fluid flowing inside a pipe) and the outer surface is a a temperature T_2 , while it is bathed by a fluid at a temperature $T_{\infty,2}$ (the air surrounding a pipe, or the fluid in the shell side – outside the tubes – of a heat exchanger).

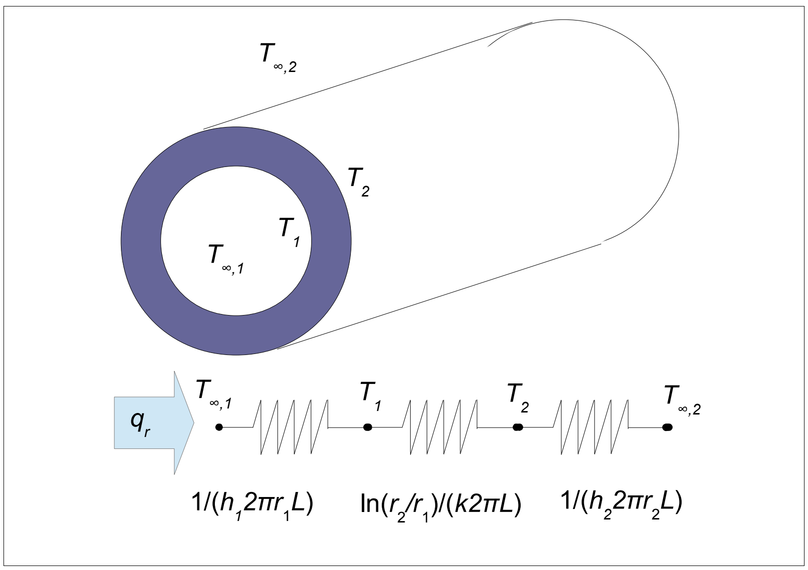


Figure 3-1. Hollow cylinder of length L with convection conditions at the inner and outer surfaces.

It can be shown that the heat transfer rate in the radial direction in a cylindrical shell is:

$$q_r = \frac{2\pi L k (T_1 - T_2)}{\ln(r_2/r_1)} \quad \textbf{(cylinder)} \tag{3-1}$$

and therefore the thermal resistance for radial conduction is

$$R_{th,\text{cond}} = \frac{\ln(r_2/r_1)}{2\pi L k} \quad \textbf{(cylinder)} \tag{3-2}$$

The heat transfer rate for the configuration of Figure 3-1 can also be written in terms of the overall temperature difference, but we would need to include the convective resistances:

$$q_r = \frac{\left(T_{\infty,1} - T_{\infty,2}\right)}{\frac{1}{2\pi r_1 L h_1} + \frac{\ln\left(r_2/r_1\right)}{2\pi k L} + \frac{1}{2\pi r_2 L h_2}} \tag{3-3}$$

This result for the heat transfer rate can also be written in terms of a U-factor, or an overall heat transfer coefficient:

$$q_r = U_1 A_1 \left(T_{\infty,1} - T_{\infty,2}\right) \tag{3-4}$$

where $A_1 = 2\pi r_1 L$ is the inside surface area, and

$$U_1 = \frac{1}{\frac{1}{h_1} + \frac{r_1}{k}\ln\frac{r_2}{r_1} + \frac{r_1}{r_2}\frac{1}{h_2}} \tag{3-5}$$

is the overall heat transfer coefficient based on the internal surface area. One could use $U_2 A_2$ in Equation (3-4) where:

$$U_2 = \frac{1}{\frac{r_2}{r_1}\frac{1}{h_1} + \frac{r_2}{k}\ln\frac{r_2}{r_1} + \frac{1}{h_2}} \tag{3-6}$$

is the overall heat transfer coefficient based on the outer surface area. Note that $U_1 A_1 = U_2 A_2$.

A review of equation (3-3) shows that two competing effects arise as the radius of insulating material r_2 increases. As r_2 increases the thermal resistance due to conduction increases, but the resistance due to convection decreases (because the outer surface area increases). This suggests that there exists a critical value of r_2 where the heat transfer rate could be a maximum. Heat transfer textbooks show this is actually the case. The **critical insulation radius** is the radius of insulating material that maximizes heat transfer; that is, below which q_r increases with increasing r and above which q_r decreases with increasing r. It can be shown that the critical insulation radius is:

$$r_{cr} = \frac{k}{h} \tag{3-7}$$

The critical insulation radius is actually desirable for electrical current flow through a wire, since the addition of electrical insulation would aid in transferring heat dissipated in the wire to the surroundings.

Conversely, if $r > r_{cr}$ any addition of insulation increases the total resistance (and therefore decrease the heat loss). This is desirable whenever it is desired to reduce heat loss to the surroundings.

This problem of potentially reducing the total resistance through the addition of insulation exists only for small diameter wires or tubes and for small convection coefficients, such that $r < r_{cr}$. For a typical conductivity of insulating material of $k = 0.02(\text{Btu/h})/(\text{ft}\,°\text{F})$ and the typical coefficient for natural convection in air of $h = 1.8(\text{Btu/h})/(\text{ft}^2°\text{F})$ we get r_{cr} of roughly $9/64^{\text{ths}}$ of an inch. So, in typical applications $r > r_{cr}$ and we need not be concerned with the critical insulation radius.

For a **spherical shell or wall** of inner radius r_1 at T_1 , and outer radius r_2 at T_2 it can be shown that the heat transfer rate in the radial direction in a spherical shell is:

$$q_r = \frac{4\pi k(T_1 - T_2)}{(1/r_1) - (1/r_2)} \quad \textbf{(sphere)} \tag{3-8}$$

and therefore the thermal resistance for radial conduction is

$$R_{th,\text{cond}} = \frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \quad \textbf{(sphere)} \tag{3-9}$$

Extended Surfaces (Fins)

Fins are extensions of a solid surface that penetrate the fluid field in order to enhance the transfer of heat by virtue of an increased surface area of contact. Some simple examples are shown in Figure 3-2.

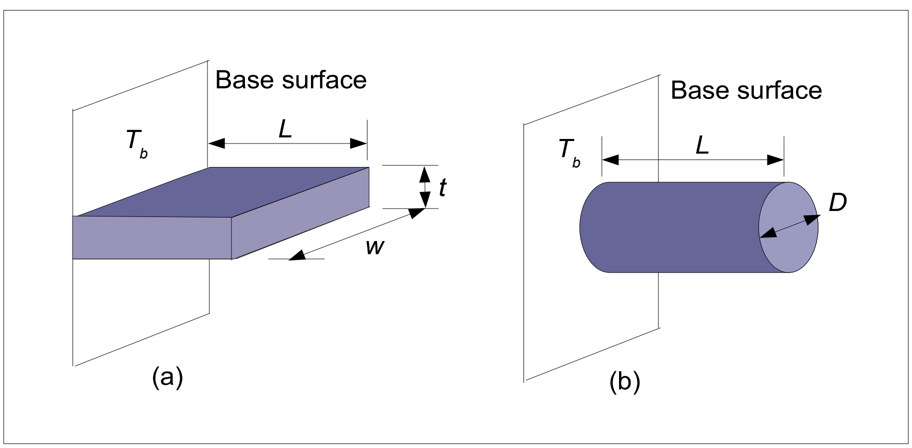


Figure 3-2. Simple straight fin configurations: (a) Rectangular and (b) Pin

Heat transfer occurs by conduction within the fin, and by convection from the surfaces of the fin. Consider a fin (made of a material of thermal conductivity k) that is bathed by a fluid of temperature T_∞ with heat transfer coefficient h . If we assume the fin is "**infinitely long**" (that is, if the temperature at the tip of the fin T_b is – for all practical purposes – equal to that of the fluid) then we could show that the **fin heat rate** q_f is given by:

$$q_f=\sqrt{h\,P\,k\,A_c}\left(T_b-T_\infty\right) \qquad \text{(straight, “infinite” fins)} \qquad (3\text{-}10)$$

where P and A_c are the cross-sectional perimeter and area, respectively. The fin heat rate is the heat transfer rate by conduction through the base of the fin. It can also be shown that the temperature along an infinite fin decays exponentially from T_b at the base ($x=0$) to T_∞ at the tip, as follows:

$$T(x)=T_\infty+\left(T_b-T_\infty\right)\exp(-m\,x) \qquad (3\text{-}11)$$

where m is referred to as the **fin parameter**, and is defined as:

$$m=\sqrt{\frac{h\,P}{k\,A_c}} \qquad (3\text{-}12)$$

There are other results available for the temperature distribution and fin heat rate in the literature for more complicated cases, such as fins with convective heat transfer at the tip, fins with a prescribed temperature at the tip, fins with varying cross-sectional area, or arrays of fins, etc. The resulting equations for these cases, however, are too complicated and are beyond the scope of the P.E. exam.

The point of using fins is to enhance heat transfer by increasing the surface area. However, the fin also introduces a conductive resistance, so the designer has to ensure the conduction resistance isn't too large. To ensure the fin isn't making matters worse, we calculate the **fin effectiveness**:

$$\epsilon_f=\frac{q_f}{h\,A_c\left(T_b-T_\infty\right)} \qquad (3\text{-}13)$$

which is the ratio of the fin heat rate to the heat transfer rate that would exist without the fin, so we want this value to be as large as possible. Typically, the fin is desired if ϵ_f is greater than about 2 and if the added costs can be justified. The fin resistance is another parameter used to quantify fin performance, and it is useful when representing a finned surface as a thermal circuit element. If we interpret $(T_b - T_\infty)$ as a driving potential for the fin heat rate q_f then, the **fin resistance** is defined as $R_{th,fin} = (T_b - T_\infty)/q_f$, which for a straight fin (like those of Figure 3-2) is:

$$R_{th,fin} = \frac{1}{\sqrt{h P k A_c}} \qquad \text{(straight, “infinite” fins)} \qquad (3\text{-}14)$$

The highest possible fin heat rate is that which would exist if the entire fin surface were at the base temperature: $q_{max} = h A_f (T_b - T_\infty)$, where A_f is the total surface area of the fin. But, since all fins have a finite conduction resistance, a temperature gradient must exist along the fin and this maximum rate q_{max} is an idealization. So, it makes sense to define the **fin efficiency** as the ratio of the actual fin heat rate to the theoretical, maximum heat rate:

$$q_{max} = h A_f (T_b - T_\infty) \qquad (3\text{-}15)$$

PROBLEMS

03-01. A schedule 40, 1-inch seamless stainless steel pipe (thermal conductivity = 10 (Btu/h)/(°F ft); OD=1.315 inch; wall thickness=0.133 inches) carries a chilled brine solution. The solution inside the pipe is at 40°F with a convective coefficient of 5 (Btu/h)/(°F ft^2) while ambient air in the facility is at 75°F and convection outside the pipe is characterized by a heat transfer coefficient of 1 (Btu/h)/(°F ft^2).

(a) Calculate the heat transfer rate per unit length of pipe, in (Btu/h)/ft.

(b) Calculate the outer surface temperature of the pipe, in °F.

(c) Calculate the heat transfer rate per unit length, if a ½-inch thick layer of calcium silicate insulation with conductivity 0.03 (Btu/h)/(°F ft) is added to the pipe.

(d) Calculate the outer surface temperature of the pipe, and the outer surface temperature of the insulation layer.

