SLAY the P.E.

www.SlaythePE.com

MECHANICAL ENGINEERING
THERMAL AND FLUID SYSTEMS
STUDY PROBLEMS

FLUID MECHANICS WITH
HYDRAULICS & FLUID APPLICATIONS

How to use this book

The exam specifications in effect since April 2017 state that approximately 6 problems from the "Fluid Mechanics" topic will be in the morning "Principles" portion of your exam. The specifications also state that approximately 24 problems comprise the "Hydraulic and Fluid Applications" portion of the exam. You can expect, therefore, that Fluid Mechanics with Hydraulic and Fluid Applications will occupy roughly 30 of the 80 problems (almost 40%) in the PE Mechanical Thermal-Fluid Systems Exam. The problems in this book are designed to help you prepare for both of these broad topics. Reviewing all the problems in this book will prepare you for these questions by focusing on exam-like problems, recognizing the common pitfalls, and developing a solid understanding of the underlying theories.

How it works

This study problems book works on what we call the "principle of progressive overload". With this technique you start with very easy problems and smoothly progress towards more complex problems. A good example of progressive overload is the story of the famous wrestler Milo of Croton in ancient Greece. This extraordinarily strong man was allegedly capable of carrying a fully grown bull on his shoulders. He was reported to have achieved this tremendous strength by walking around town with a new born calf on his shoulders every single day. As the calf grew, so did the man's strength.

We recommend you work the problems in this book in the order they are presented. Within each section of the book, the first problems will feel "light", like carrying that baby calf – you might even be tempted to skip them. We strongly urge you to resist this temptation. As you progress, the problems become harder, but the work you've been putting in with all the previous problems will bear fruit. You will be pleasantly surprised at how relatively easy those "hard" problems will seem. You will soon be carrying intellectual bulls on your shoulders! **The problems that are considered "exam-level difficult" are denoted with an asterisk**.

This book is comprised of the following sections:

For the most part, these sections are not independent and build from the previous ones. We recommend you go through them in the order presented, and be sure to review them all. Each section begins with a brief discussion of the relevant concepts and equations. These discussions are laser-focused on the aspects that are relevant to the P.E. exam and do not go into derivations with academic rigor.

This page was intentionally left blank

SECTION 01: Basic Information and Properties of Fluids

Physical quantities require quantitative descriptions when solving engineering problems. Consider the density as one such quantity. It is a measure of the mass contained in a unit volume. However, density is not considered a "fundamental" dimension. Only length, mass, time, temperature plus five more[1] are fundamental. All other quantities can be expressed in terms of fundamental dimensions. For instance, the dimensions of force can be related to the fundamental dimensions of mass, length, and time. To give the dimensions of a quantity a numerical value, a set of units must be selected.

All equations must be **dimensionally homogeneous**. That is, every term in an equation must have the same units. If, at some stage of an analysis, you find yourself in a position to add two quantities that have different units, it is a clear indication that something went wrong at an earlier stage. So checking dimensions can serve as a valuable tool to spot errors. We strongly encourage you to always keep track of units and never, ever, write down a number without its accompanying units. Lack of experience and carelessness with units and unit conversions is one of the biggest hindrances towards success with the P.E. exam.

Unity conversion ratios are identically equal to 1 and are unit-less, and thus such ratios (or their inverses) can be inserted conveniently into any calculation to properly convert units, because any quantity multiplied (or divided) by 1 remains unchanged. For example, the quantity:

$$\left|\frac{1\,\text{lbf}}{\left(32.174\,\frac{\text{lbm}\cdot\text{ft}}{\text{s}^2}\right)}\right|$$

is a ratio of two quantities that are identical, so it is equal to 1. Likewise, quantities such as:

$$\left|\frac{12\,\text{in}}{1\,\text{ft}}\right| \qquad \left|\frac{7.48052\,\text{gallons}}{1\,\text{ft}^3}\right| \qquad \left|\frac{6.89476\,\text{kPa}}{1\,\text{psi}}\right|$$

are all identically equal to 1 and are frequently inserted into calculations to ensure the dimensional homogeneity of equations. Some books insert the archaic gravitational constant g_c defined as $g_c = 32.174\,\text{lbm}\cdot\text{ft}/\left(\text{lbf}\cdot\text{s}^2\right)$ into equations in order to force units to match. This practice leads to unnecessary confusion and is strongly discouraged. We recommend you instead use unity conversion ratios.

1 The others are electric current, luminous intensity, plane angle, solid angle, and amount of substance.

When we say that equations must be dimensionally homogeneous, we mean that the dimensions of the left side of the equation must be the same as those on the right side, and all additive separate terms must have the same dimensions. We accept as a fundamental premise that all equations describing physical phenomena must be dimensionally homogeneous. If this were not true, we would be attempting to equate or add unlike physical quantities, which would not make sense. For example, the equation for the velocity, V, of a uniformly accelerated body is:

$$V = V_0 + a t$$

Here we note that the dimensions of all three terms are length/time – thus, the equation is dimensionally homogeneous. To illustrate the use of unity conversion ratios consider this example (formatted in "PE style"):

EXAMPLE. The velocity of a uniformly accelerated body is given by $V = V_0 + a t$. Therefore, the velocity (inches per second) at $t = 0.25\,\text{min}$ if the initial velocity $V_0 = 50\,\text{ft/min}$ and $a = 0.85\,\text{ft/s}^2$ is most nearly:

(A) 50.85
(B) 163
(C) 815
(D) 9780

The first way the careless test-taker messes this one up is by not writing down the units and doing the following:

$$V = V_0 + a t = 50 + 0.85 \times 1 = 50.85$$

This is of course terribly wrong, but people actually do this! As you can see, 50.85 is one of the answer choices. So, careless test-taker #1 would select (A) 50.85, and be wrong. The actual test will have "distractor" answers like this. Be careful.

Careless test taker #2 does a little bit better, and actually gets a "correct" answer as follows:

$$V = V_0 + a t = 50\frac{\text{ft}}{\text{min}} + 0.85\frac{\text{ft}}{\text{s}^2} \times 0.25\,\text{min}$$

$$= 50\frac{\text{ft}}{\text{min}} + 0.85\frac{\text{ft}}{\text{s}^2} \times \left|\frac{60\,\text{s}}{1\,\text{min}}\right|^2 \times 0.25\,\text{min} = 50\frac{\text{ft}}{\text{min}} + 765\frac{\text{ft}}{\text{min}} = 815\frac{\text{ft}}{\text{min}}$$

Note the use of the unity conversion factor on the second term, to ensure dimensional homogeneity of the calculation. Note that 815 is one of the answer choices. However, the question specifies the units of the answer must be in inches per second. Choosing (C) 815, would be wrong – even though the answer is correct! We leave it as an exercise for you to confirm that the correct answer is (B).

Some valid equations contain constants with dimensions. For example, the equation for the distance, d, traveled by a body in free-fall can be written as:

$$d = 16.1\, t^2$$

and a check of the dimensions reveals that the constant 16.1 must have the dimensions of length over time squared, if the equation is to be dimensionally homogeneous. This equation is actually a particular case of the more general equation from physics for freely falling bodies:

$$d = g\frac{t^2}{2}$$

in which g is the acceleration of gravity. This equation is dimensionally homogeneous and valid in any system of units, but for For $g = 32.2\ \mathrm{ft/s^2}$ the equation reduces to $d = 16.1\, t^2$ which is valid only for the system of units using feet and seconds.

Pressure results from compressive forces acting on an area of a continuous medium. Pressure is a scalar function; it acts equally in all directions at a given point for a fluid whether it is quiescent or in motion.

The **absolute pressure** reaches zero when an ideal vacuum is achieved (i.e., in a space where there are no molecules) thus negative absolute pressures are not possible. In addition to the absolute pressure scale, pressures can be measured with respect to the local atmospheric pressure. The term **gage-pressure** refers to values of pressure measured relative to the local atmospheric value. The gage-pressure is negative whenever the absolute pressure is lower than the local atmospheric pressure. Negative gage-pressure values are referred to as **vacuum** pressures. The following equation provides the conversion from gage to absolute pressures:

$$p_{\text{absolute}} = p_{\text{gage}} + p_{\text{atm}} \tag{1-1}$$

In the International System of Units (SI), pressure is measured in **Pascals** (Pa) with the SI prefixes kilo-, Mega-, and Giga-, being among the most used in ordinary engineering applications.

$$1\ \text{kPa} = 10^3\ \text{Pa}$$
$$1\ \text{MPa} = 10^3\ \text{kPa} = 10^6\ \text{Pa}$$
$$1\ \text{GPa} = 10^3\ \text{MPa} = 10^6\ \text{kPa} = 10^9\ \text{Pa}$$

The **bar** is a unit of pressure of the “metric system”, but is not formally approved as part of the SI. It is defined as 100 kPa, or 0.1 MPa. In the US Customary System (USCS) of units, pressure is typically expressed in pounds-force per square inch (lbf/in^2, or **psi**) or pounds-force per square foot (**psf**).

In general, one of the words “absolute” or “gage” typically follows a value if the given pressure is given as an absolute or gage pressure, respectively (e.g., “ $p=750\,\text{kPa}$ absolute”) but sometimes it is specified in the units themselves, especially in the USCS. For example $p=580\,\text{psia}$ refers to an absolute pressure of 580 pounds-force per square inch, and “ $p=3.5\,\text{psig}$ ” refers to a gage pressure of 3.5 pounds force per square inch.

The local atmospheric pressure varies with elevation, and the “standard” value at sea level is defined as 101.3 kPa, 14.7 psi, 30 in. Hg, 760 mmHg, or 1.013 bar.

A fluid's **density**, ρ ,is defined as the mass of a unit of volume, and the **specific volume**, γ ,is defined as the weight of a unit of volume:

$$\gamma=\rho\, g \tag{1-1}$$

where g is the local gravity. The units of specific weight are N/m^3 and lbf/ft^3. The specific gravity SG is often used to determine a fluid's density or specific weight. It is defined as the ratio of the density of a substance to that of water at a reference temperature:

$$SG=\frac{\rho}{\rho_{\text{water, std}}}=\frac{\gamma}{\gamma_{\text{water, std}}} \tag{1-2}$$

If the reference temperature is the typically used value of 4°C then:

$$\rho_{\text{water, std}}=1000\,\frac{\text{kg}}{\text{m}^3}=62.4\,\frac{\text{lbm}}{\text{ft}^3}=0.0361\,\frac{\text{lbm}}{\text{in}^3} \qquad \gamma_{\text{water, std}}=9810\,\frac{\text{N}}{\text{m}^3}=62.4\,\frac{\text{lbf}}{\text{ft}^3}=0.0361\,\frac{\text{lbf}}{\text{in}^3}$$

For fluids at rest, it can be shown that the pressure does not vary in the horizontal plane (which is defined as the plane normal to the gravity vector), and if we assume the density is constant then:

$$\frac{p}{\gamma}+z=\text{constant} \tag{1-3}$$

where z is positive in the vertically upward direction, so that pressure increases with depth. The quantity $(p/\gamma)+z$ is known as the **piezometric head**. If we define as $z=0$ the location of an interface between the liquid and air above it (this is known as a free surface) at atmospheric pressure, then $p=0$ (gage) at $z=0$, thus:

$$p=\gamma\, h \tag{1-4}$$

where h is the depth below the free surface. Equation (1-4) is useful in converting pressure to an equivalent height of liquid. For example, atmospheric pressure is often expressed in millimeters of mercury – this means that the atmospheric pressure is equal to the pressure at a certain depth in a mercury column, and by knowing the specific weight of mercury, we can determine the depth using equation (1-4).

Manometers are instruments that use columns of liquid to measure pressures. Figure 1-1 shows a U-tube manometer, which is used to measure relatively small pressures. In the figure, point "1" is located in the center of a pipe that extends in the direction perpendicular to the page, and point "2" is at the free surface of the right column, exposed to atmosphere.

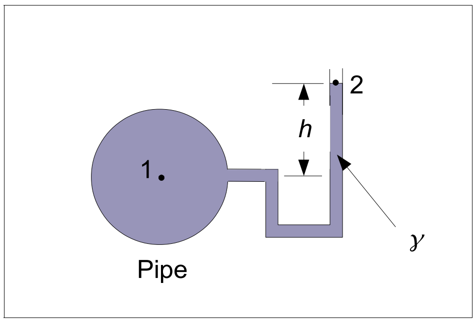


Figure 1-1: U-tube Manometer.

We can use equation (1-3) to write:

$$p_1 + \gamma z_1 = p_2 + \gamma z_2$$

where the datum (i.e., the location where $z=0$) can be located anywhere we want. We know also that $p_2=0$ psig (we could also use $p_2=p_{\text{atm}}$ if we wish to work with absolute pressures). Choosing now $z_1=0$, the above expression yields: $p_1=\gamma h$.

Figure 1-2 shows a manometer that could be used to measure larger pressures since we could choose a fluid with a large specific weight, γ_B (for example if fluid 1 is water and fluid 2 – the red one – is mercury, then $\gamma_B = 13.6\,\gamma_A$). The pressure at location 1 can be determined by defining 4 points as shown in Figure 1-2.

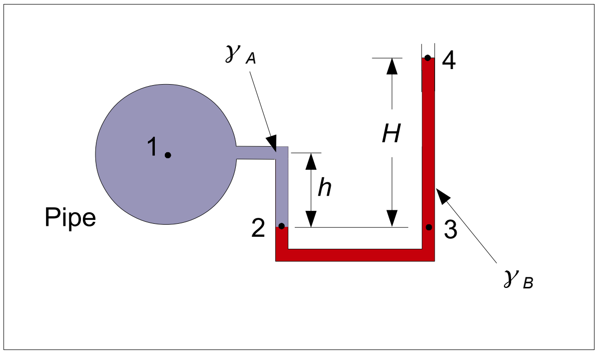


Figure 1-2: U-tube Manometer with dedicated manometer fluid

We can use equation (1-3) to write:

$$p_1 + \gamma_A z_1 = p_2 + \gamma_A z_2$$

$$p_3 + \gamma_B z_3 = p_4 + \gamma_B z_4$$

but, $p_2 = p_3$ because points 2 and 3 are on the same horizontal plane ($z_2 = z_3$) and within the same fluid. We can add the above equations to obtain:

$$p_1 + \gamma_A (z_1 - z_2) = \gamma_B (z_4 - z_3) + p_4$$

and we can set $p_4 = 0$ and work with gage pressures. The resulting expression for p_1 is:

$$p_1 = \gamma_B H - \gamma_A h$$

Note that if fluid A (the one in the pipe) is a gas, and fluid B is a liquid, it generally follows that $\gamma_A \ll \gamma_B$ and the second term in the above expression can be safely neglected.

The lay person's understanding of viscosity is that it is related to the internal stickiness of a fluid. In more technical terms, **viscosity** is linked to the rate of deformation of a fluid under shear stress. For a given applied shear stress, a high viscosity fluid will experience smaller deformation than a low viscosity fluid under the same stress.

Consider the fluid within a small gap between two concentric cylinders, as shown in Figure 1-3. An externally applied torque causes the rotation of the inner cylinder, while the outer cylinder remains stationary. The resistance to the rotation is due to the fluid's viscosity.

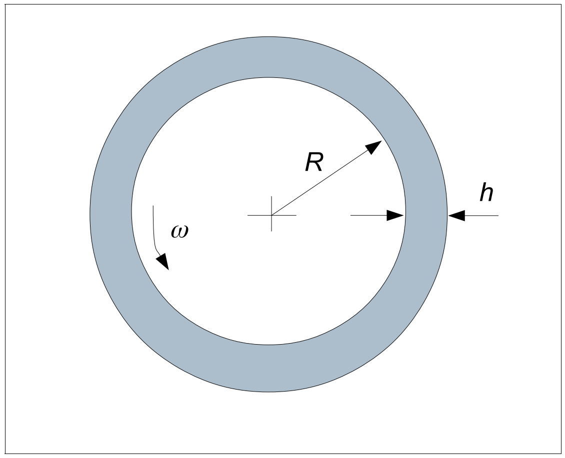


Figure 1-3: Fluid being sheared between two cylinders: Fixed outer cylinder and rotating inner cylinder.

The fluid velocity u in the thin gap varies linearly[2]. The fluid layer adjacent to the inner cylinder will have a velocity equal to the tangential velocity of the cylinder, ωR, where ω is the rotational speed of the inner cylinder (in rad/s) and R is the radius of the inner cylinder. The fluid layer adjacent to the outer (stationary) cylinder will be stationary. If the gap between the cylinders is h, then the velocity gradient du/dr across the gap is $(\omega R)/h$.

When a torque T is applied to the inner cylinder, the fluid in the gap experiences a shear stress τ, which is related to the velocity gradient:

$$\tau=\mu\left|\frac{du}{dr}\right|=\mu\frac{(\omega R)}{h}$$

Where the constant of proportionality between sheer stress and velocity gradient (in this very simple flow) is the viscosity, μ.

Since the torque is:

$$T=\text{stress}\times\text{area}\times\text{moment arm}$$

$$T=\tau\times 2\pi R L\times R$$

or,

$$T=\frac{2\pi R^3\omega L\mu}{h} \tag{1-4}$$

Additionally, the power P, transmitted by a shaft rotating with an angular velocity ω by applying a torque T is given by $P=\omega T$, so inserting that into equation (1-4) we get:

2 If the gap width h is not small relative to R the velocity variation will not be linear.

$$P = \frac{2\pi\omega^2 R^3 L\mu}{h} \tag{1-5}$$

where we have neglected the stress acting on the ends of the cylinders. Since the torque depends on the viscosity, this type of cylinder gap arrangement is used as a **viscometer** – a device used to measure viscosity. If the shear stress in a fluid is directly proportional to the velocity gradient (shear strain rate), the fluid is said to be a **Newtonian fluid**. If the shear stress varies in a non-linear way with the strain rate, then the fluid is **non-Newtonian** (liquid plastics, blood, slurries, toothpaste, etc) and depending on the functional form of this relationship the fluids are further classified as thixotropic, dilatant, Bingham plastics, rheopectic, etc. **Rheology** is the branch of physics that deals with this study of deformation and flow of materials.

From $\tau = \mu\,(du/dy)$ it can be readily deduced that the dimensions of viscosity are force×time/length^2. Thus, in the USCS of units viscosity is given as lbf·s/ft^2 and in SI units as N·s/m^2. The poise (P) is an alternatively used viscosity unit and 1 P = 0.1 N·s/m^2. The poise is often used with the prefix "centi-" because the viscosity of water at 20 °C is almost exactly 1 cP where 1 cP = 10^{-3} N·s/m^2.

The group μ/ρ appears often in the derivation of equations, so it has become customary to give it a name, **kinematic viscosity**:

$$\nu = \frac{\mu}{\rho} \tag{1-6}$$

and thus the viscosity μ is also referred to as "dynamic" viscosity. The dimensions of kinematic viscosity are length^2/time, and the USCS units are ft^2/s and SI units are m^2/s. The stoke[3] (St) is an alternatively used unit of kinematic viscosity and 1 St = 10^{-4} m^2/s. The stoke is often used with the prefix "centi-" because the kinematic viscosity of water at 20 °C is almost exactly 1 cSt (10^{-6} m^2/s).

Given a pressure change Δp applied to a fluid, a packet of fluid with volume $V\!\!\!/$ will experience a change in relative volume $(\Delta V\!\!\!/)/V\!\!\!/$. The **bulk modulus of elasticity**, B gives the relative change in volume for a given change in pressure applied to a fluid at constant temperature, T :

$$B = \left.\frac{\Delta p}{(\Delta V\!\!\!/)/V\!\!\!/}\right|_T = \rho \left.\frac{\Delta p}{\Delta \rho}\right|_T \tag{1-7}$$

For gases, it can be shown that the bulk modulus is equal to the pressure. For liquid water at standard conditions, the bulk modulus is roughly 310,000 psi – this means that to cause a 1% change in density

3 In the United States, it is more common to use the singular "Stoke", but the unit is named after George G Stokes, hence in the rest of the world it is more common to hear "Stokes" as a unit of kinematic viscosity.

(i.e., $(\Delta\rho)/\rho = 0.01$) of water, a pressure rise of 3,100 psi is required. The bulk modulus is also used to determine the **speed of sound** c in a fluid:

$$c = \sqrt{\frac{B}{\rho}} \tag{1-8}$$

which yields about 4,800 feet per second for the speed of sound in water at standard conditions.

PROBLEMS:

01-01. If V is a velocity, determine the dimensions of Z, α, and G, which appear in the dimensionally homogeneous equation $V = Z(\alpha - 1) + G$

01-02. The pressure difference, Δp, across a partial blockage in an artery is approximated by the equation:

$$\Delta p = K_v \frac{\mu V}{D} + K_u \left(\frac{A_0}{A_1} - 1\right)^2 \rho V^2$$

where V is the blood velocity, μ the blood viscosity, ρ the blood density, D the artery diameter, A_0 the area of the unobstructed artery, and A_1 the area of the blockage. Determine the SI dimensions of the constants K_v and K_u.

01-03. A pressure of 4 psig is measured at an elevation of 6,560 ft – where the atmospheric pressure is 11.5 psia. What is the absolute pressure in (a) kPa, (b) mm of Hg, and (c) ft of water?

01-04. A gage reads a vacuum of 24 kPa. What is the absolute pressure (a) at sea level, and (b) at an altitude of 4000 m where the atmospheric pressure is 0.616 bar?

01-05. Calculate the gage pressure (in psi) in a pipe transporting air if a U-tube manometer measures 9.85 inches of mercury. Note that the weight of air in the manometer is negligible.

01-06. If the air pressure in a pipe is 65 psig, what will a U-tube manometer with mercury indicate?

01-07*. The Weber number is a dimensionless parameter, given by:

$$\mathrm{We}=\frac{\rho V^2 L}{\sigma}$$

where ρ is density, V is a velocity, L is a length, and σ is a material property of the fluid. Since the Weber number is dimensionless, the units of σ must be:

(A) lbm/(ft·s)

(B) lbf/ft

(C) ft/lbf

(D) s^2/lbm

01-08*. A formula to estimate the volume rate of flow, Q , flowing over a dam of length, B , is given by the equation $Q=3.09\,B\,H^{3/2}$,where H is the depth of the water above the top of the dam. This formula gives Q in ft^3/s when B and H are in feet. An equivalent formula that gives Q in gallons per day when B and H are in feet is:

(A) $Q=0.00027\,B\,H^{3/2}$

(B) $Q=0.413\,B\,H^{3/2}$

(C) $Q=3.09\,B\,H^{3/2}$

(D) $Q=35{,}690\,B\,H^{3/2}$

01-09*. While performing a calculation, an engineer arrives at the following equation to calculate a force: $F=\dot{m}V+pA$, where $\dot{m}=600\,\mathrm{kg/min}$, $V=2\,\mathrm{m/s}$, $p=29.7\,\mathrm{kPa}$, and $A=50.3\,\mathrm{cm}^2$.

Under these conditions, the force F (N) is most nearly:

(A) 35

(B) 169

(C) 1513

(D) 2694

01-10*. While performing a calculation, an engineer arrives at the following equation to calculate a volumetric flow rate: $Q=F/(\rho V)$,where $F=35$ pounds-force, $\rho=6.26\,\text{lbm}/\text{gallon}$, and $V=15\,\text{ft}/\text{s}$. Under these conditions, the flow rate Q (cubic feet per hour) is most nearly:

(A) 0.373

(B) 12

(C) 5,775

(D) 43,172

01-11*. A heat transfer oil with a specific gravity of 0.86 flows through a pipe. The reading from a U-tube manometer is 9.5 in Hg. The oil in the manometer is depressed 5 inches below the pipe centerline, and the local atmospheric pressure is 14.7 psi. Under these conditions, the absolute pressure (psi) of the oil in the pipe at the location of the manometer is most nearly:

(A) 4.5

(B) 7.8

(C) 14.7

(D) 19.2

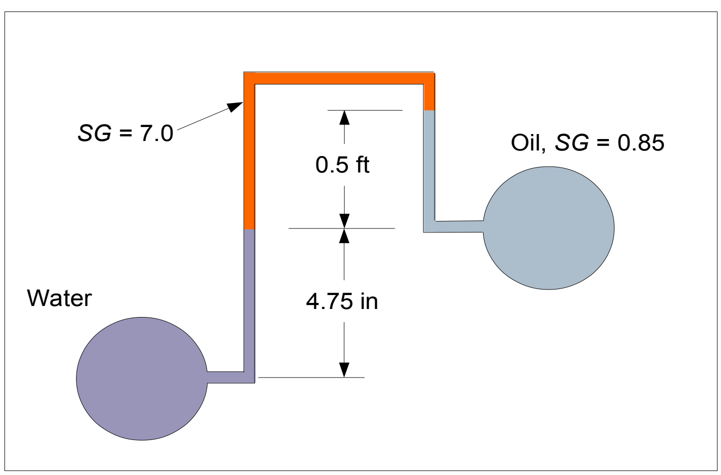
01-12*. The pressure in the water pipe is 2.5 psig. The pressure (psig) in the oil pipe is most nearly:

(A) 1.0

(B) 2.0

(C) 2.2

(D) 4.5



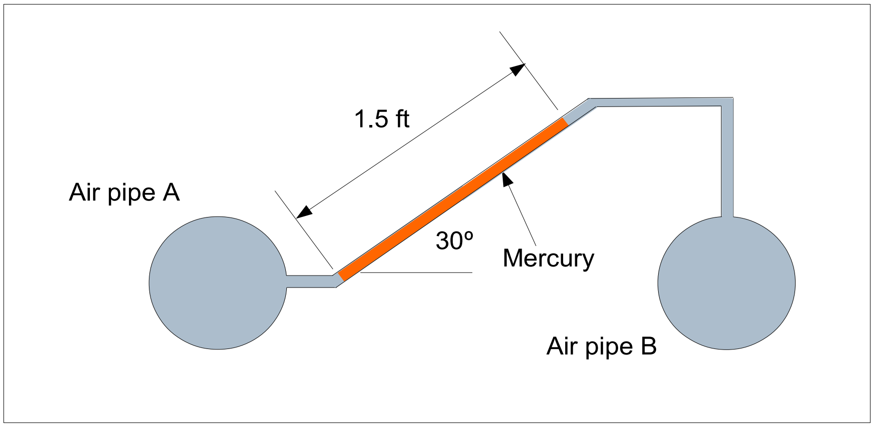
01-13*. The pressure in air pipe A is 15 psig. The pressure (psig) in air pipe B is most nearly:

(A) 6.2

(B) 10.6

(C) 14.6

(D) 15.0



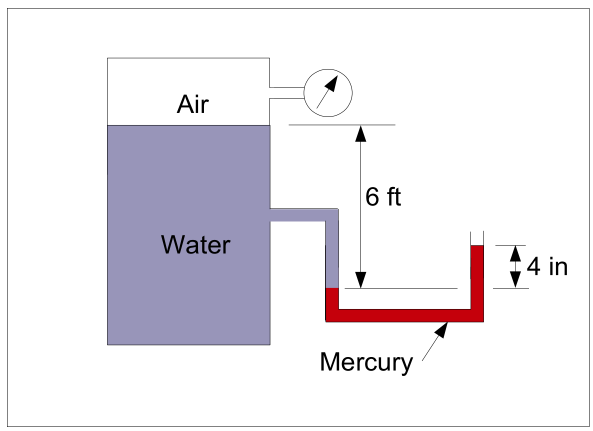
01-14*. The reading of the pressure gage in psi is most nearly:

(A) a vacuum of 2.5

(B) a vacuum of 1.6

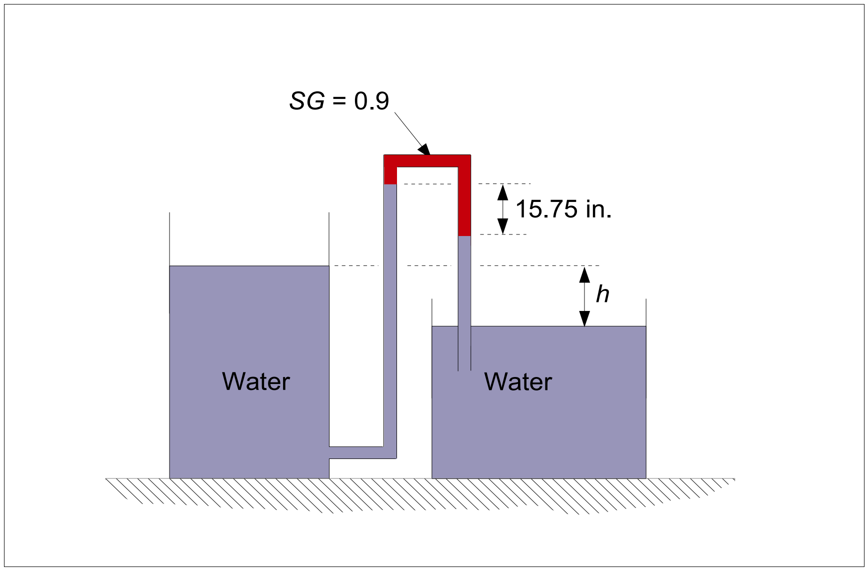
(C) a vacuum of 0.64

(D) 1.74



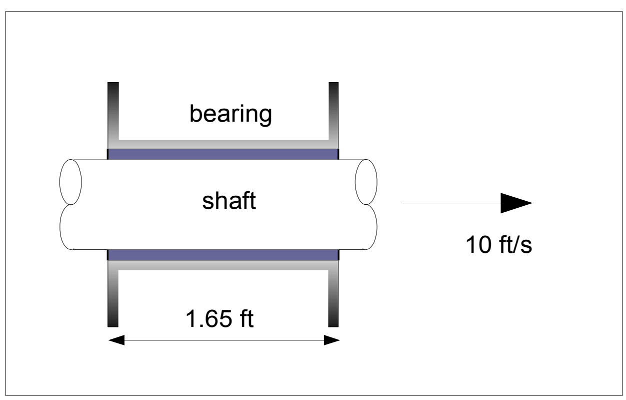
01-15*. The elevation difference h (ft) between the water levels in the two open tanks is most nearly:

(A) 0.13
(B) 1.6
(C) 2.6
(D) 3.5



01-16*. A 1 inch-diameter shaft is pulled through a cylindrical bearing as shown in the figure. The lubricant that fills the 0.012-inch gap between the shaft and bearing is an oil having a kinematic viscosity of 0.0086 ft^2/s and a specific gravity of 0.91. The force required to pull the shaft at a velocity of 10 ft/s is most nearly:

(A) 0.46
(B) 2.5
(C) 3.8
(D) 5.5



This page was intentionally left blank

Section 02: Classification of Flows, Terminology & The Bernoulli Equation

If in a given flow field the velocity vector only depends on one spatial coordinate (e.g., x, y, or z) we say it is a **one-dimensional flow**. As examples, consider the flow pattern that occurs in long, straight circular pipes or between parallel plates as shown in Figure 2-1. For the pipe flow, the velocity only depends on the radial coordinate, r, and in the parallel plates case, the velocity only depends on the vertical coordinate y.

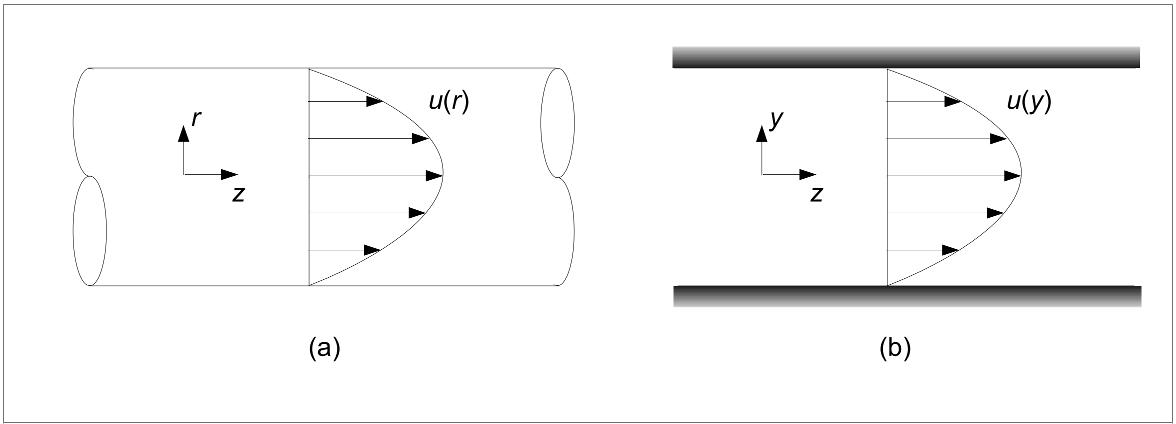


Figure 2-1: One-dimensional flow: (a) flow in a pipe, (b) flow between parallel plates

In the cases above, even if the flow is unsteady (i.e. time-dependent, as in a startup or shutdown) the flow would still be one-dimensional. The velocity profiles shown in Figure 2-1 can also be referred to as **fully developed** flows, which means that the velocity profiles do not change in the direction of flow (i.e., the z direction in the figure).

Many times in engineering applications considerable simplicity is achieved by invoking the assumption of **uniform flow**, which implies that the velocity is constant over the cross-sectional area of the pipe or conduit. The average velocity may change from one section to the other, but the flow conditions only depend on the space variable in the direction of flow – as in Figure 2-2.

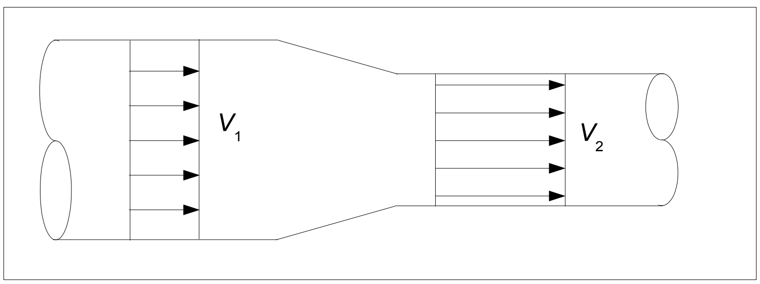


Figure 2-2: Uniform velocity profiles

In a broad sense, a fluid flow may be classified as inviscid or viscous. An **inviscid flow** is one in which the effects of the fluid's viscosity does not significantly influence the flow field, so these effects are neglected. The primary class of flows that can be accurately analyzed as inviscid are **external flows**, that is, flows that exist exterior to a solid body such as flow around an airfoil or a vehicle. Any viscous effects that may exist are all within a very thin region adjacent to the surface of the solid body known as a **boundary layer**. In many practical applications, the boundary layers are so thin that they can be ignored when studying the gross, macroscopic features of a flow. **Viscous flows** include **internal flows**, such as flows in pipes, conduits, and open channels. Viscous effects are responsible for the "losses" encountered in these flows which we will examine in greater detail in further sections.

Viscous flows can be further classified as laminar or turbulent. In **laminar flow**, there is no significant mixing of fluid particles with their neighboring particles. Imagine a set of "sheets" sliding past each other. We recommend googling "laminar flow visualization" videos for some impressive demonstrations of this feature. In many of these videos, you'll see a dye being injected in the flow. The dye does not mix, retaining its "identity" for a relatively long time, over relatively long distances. The flow may be time-dependent, or it may be steady. In a **turbulent flow**, quantities such as velocity and pressure show random fluctuations with time and with the space coordinates. For turbulent conditions, a flow is deemed "steady" if the time-averaged physical quantities (e.g., pressure, velocity, temperature, etc) do not change in time. If you were to measure the velocity magnitude of a steady, turbulent flow in a given location, you would notice an irregular pattern made by all the instantaneous values, but they would all be "around" an average value that doesn't change with time. A dye injected into a turbulent flow would quickly mix by the action of randomly moving particles.

The interaction of three key parameters defines if a fluid flow is laminar or turbulent. These three parameters are the length scale, the fluid velocity scale, and the fluid's viscosity. The three parameters are combined into a single dimensionless parameter (that can serve as a tool to make predictions about the flow being laminar or turbulent) known as the **Reynolds number**:

$$\mathrm{Re}=\frac{V\,L}{\nu} \tag{2-1}$$

where V and L are a characteristic velocity and length, respectively, and ν is the fluid's kinematic viscosity. If the Reynolds number is relatively low, the flow will be laminar; if it is large, the flow will be turbulent. This phenomenon is quantified with the use of a **critical Reynolds number**, Re_{crit} , so that the flow is laminar if $\mathrm{Re} < \mathrm{Re}_{\mathrm{crit}}$. The magnitude of $\mathrm{Re}_{\mathrm{crit}}$ depends on the flow configuration, for example for flow inside a rough walled pipe in most engineering applications, a value $\mathrm{Re}_{\mathrm{crit}} = 2{,}000$ is typically used. A simple example of a transition from laminar to turbulent may be observed on the smoke rising from a cigarette or a smoke stack. For a certain distance, the smoke rises in a smooth (laminar) manner but then abruptly, the smoke mixes up and becomes turbulent and the narrow smoke column widens and diffuses.

Finally, another flow classification distinguishes flows that are compressible from those that are incompressible. An **incompressible flow** exists if the density of each fluid particle remains relatively constant as it moves through the flow field. For most practical purposes, liquid flows are incompressible. Also, under certain conditions involving minor pressure changes and low velocities, gas flows may be considered incompressible. If the gas experiences relatively large density changes which influence the flow, then it is **compressible** flow. For compressible flow, there is a great distinction between flows involving velocities less than that of sound (**subsonic** flows) and flow involving velocities greater than that of sound (**supersonic** flows).[4] The **Mach number** is a measure of the flow characteristic velocity relative to the local speed of sound:

$$M = \frac{V}{c} \tag{2-2}$$

where V is the gas speed and c is the local speed of sound. Thus, when $M < 1$ the flow is subsonic, and when $M > 1$ the flow is supersonic. In gases, the speed of sound is given by $c = \sqrt{k\,RT}$ where T is the absolute temperature, k is a property known as the specific heat ratio, and R is the particular gas constant.[5] For air $k = 1.4$, and the gas constant is:

$$R_{air} = \begin{Bmatrix} 53.34\,\mathrm{ft \cdot lbf / lbm \cdot °R} \\ 0.3704\,\mathrm{psia \cdot ft^3 / lbm \cdot °R} \\ 640.08\,\mathrm{psia \cdot in^3 / lbm \cdot °R} \\ 1{,}716.2\ \mathrm{ft^2 / s^2 \cdot °R} \\ 0.0686\,\mathrm{Btu / lbm \cdot °R} \\ 0.287\,\mathrm{kJ / kg \cdot K} \end{Bmatrix}$$

When $M < 0.3$ it can be shown that density variations are not important so $\mathrm{M} < 0.3$ (which corresponds roughly to a velocity under 300 ft/s or 100 m/s for standard air) is typically used as a criterion to decide

4 Sonic speed in air at 70°F (21.1°C) is about 1,128 ft/s (343.9 m/s)
5 For a thorough review of the ideal gas law please consult our Thermodynamics and Energy Balances ebook.

to model flow as incompressible without any significant loss of accuracy.

The well-known **Bernoulli equation**[6] adopts the following form when applied between two points on the same streamline:

$$\frac{V_1^2}{2}+\frac{p_1}{\rho}+g\,z_1=\frac{V_2^2}{2}+\frac{p_2}{\rho}+g\,z_2 \tag{2-3}$$

If we divide the equation above by g, it becomes:

$$\frac{V_1^2}{2g}+\frac{p_1}{\gamma}+z_1=\frac{V_2^2}{2g}+\frac{p_2}{\gamma}+z_2 \tag{2-4}$$

The term $V^2/2\,g$ is known as the **velocity head** and it represents the vertical distance needed for the fluid to fall freely (neglecting friction) if it is to reach velocity V from rest. The term $p/\,\gamma$ is known as the **pressure head**, and it represents the height of a column of the fluid that is needed to produce the pressure p . The elevation term z is related to the potential energy of the fluid and it is known as the **elevation head**. The sum $(p/\,\gamma)+z$ is called **piezometric head**, and the sum of all three terms is the **total head**. The pressure is sometimes called **static pressure** and the group

$$p_T=p+\rho\,\frac{V^2}{2} \tag{2-3}$$

is called the **total pressure**, or **stagnation pressure**. Consider the arrangement shown in Figure 2-3. Here we see a pressure gage (or **piezometer**) mounted on the wall of a pipe or conduit, measuring the static pressure at location 1. The probe on the right is a pitot-tube which is a thin, dead-end conduit inserted in the flow. The fluid inside the tube (point 2) is stagnant.

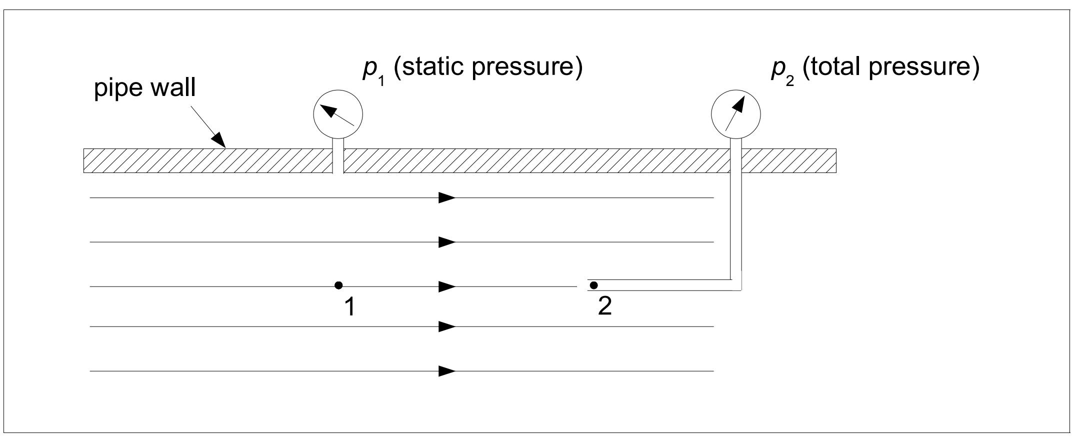


Figure 2-3: Pressure probes: piezometer (for static pressure) and pitot tube (for total pressure).

6 This equation is obtained by applying F = ma along a streamline for steady, inviscid, incompressible flow.

If we apply Equation (2-3) from point 1 to point 2 we see that the pressure read by the pitot probe is the stagnation pressure. The difference between the readings of the two probes can be used to determine the velocity at point 1. Another device, known as a **pitot-static probe** combines both devices into one instrument. The pitot-static probe provides the value of $p_2 - p_1$.

As an exercise, apply the Bernoulli equation between points 1 and 2 in Figure 2-3 to show that the velocity at point 1 is given by:

$$V_1 = \sqrt{\frac{2}{\rho}(p_2 - p_1)}$$

The Bernoulli equation can be used on many steady flow situations of engineering interest, as long as viscous and compressibility effects are not important. Examples of external flows where the Bernoulli equation is typically applied include determining the height of a jet of water, the force applied on windows due to wind, or the surface pressure on a low velocity airfoil.

Examples of internal flows where the Bernoulli equation is typically applied involve internal flows over short distances such as flow through contractions and flow from a plenum, nozzles and in pipes of variable diameter for which the fluid velocity changes because the flow area is different from one section to another. For these situations it is necessary to use the concept of conservation of mass (the continuity equation) along with the Bernoulli equation. For a control volume in steady state, **conservation of mass** requires that the rate at which the mass of fluid flows into the volume must equal the rate at which it flows out of the volume (otherwise, mass would not be conserved). The mass flow rate $\dot{m}$ (lbm/h, lbm/min, kg/h, etc) is given by $\dot{m} = \rho Q$ where Q is the volume flow rate (gal/min, ft^3/min, m^3/s, etc). If at a given section the cross-sectional area is A, and the flow occurs across this area (normal to the area) with an average velocity V the volume flow rate can be shown to be given by $Q = AV$. If we label the inlet to the control volume as "1" and the outlet as "2", then conservation of mass requires: $\rho_1 A_1 V_1 = \rho_2 A_2 V_2$. If the density remains constant, then $\rho_1 = \rho_2$ and the above becomes the continuity equation for incompressible flow:

$$A_1 V_1 = A_2 V_2 \tag{2-4}$$

Refer to Figure 2-2. Here we see that a reduction in the cross sectional area brings about an increase in the velocity.

PROBLEMS:

02-01. A 4-in. ID pipe carries 300 gpm of water at a pressure of 30 psig. Determine – in feet of water – (a) the pressure head, (b) the velocity head, and (c) the total head with reference to a datum plane 20 ft below the pipe.

02-02. Water flows at a rate of 300 gpm in a 4-inch, schedule-40, seamless steel pipe (ID=4.026 inches). The pipe is reduced down to a 3-inch, schedule-40, seamless steel pipe (ID=3.068 inches). What is the percentage change in velocity head after the reduction? What is the percentage change in pressure head after the reduction?

02-03*. A pitot tube measures the velocity of a small aircraft flying at an altitude of 3,000 ft., where atmospheric pressure and temperature are 13.2 psia and 48°F respectively, and the air density is 0.00218 slugs/ft^3. The pitot tube reading is 0.7 psi. Under these conditions, the speed of the aircraft (miles per hour) is most nearly:

(A) 192

(B) 282

(C) 880

(D) 921

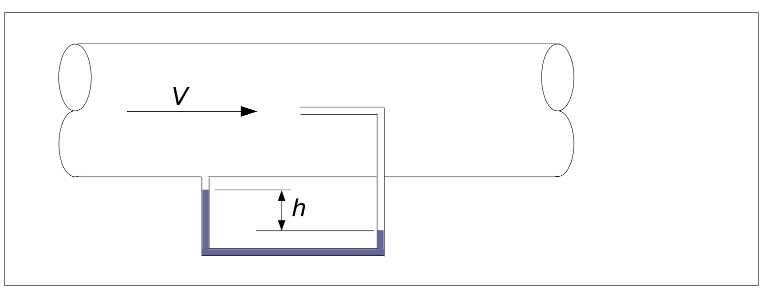
02-04*. The velocity (feet per second) of the water in the pipe, when h=4 inches of Hg is most nearly:

(A) 0.25

(B) 1.42

(C) 3.0

(D) 17.1



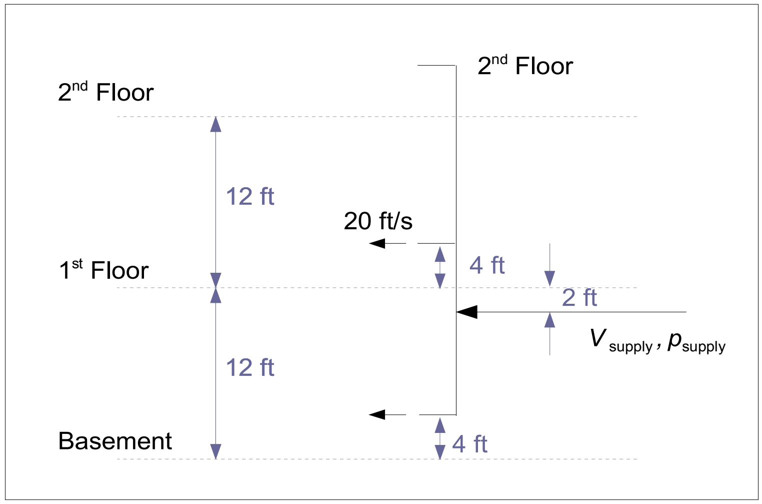
02-05*. Water is supplied to a house at pressure p_{supply} while flowing at a velocity V_{supply}. The maximum velocity of water coming out of a faucet in the first floor is 20 ft/s. Neglect any friction losses (e.g., assume inviscid flow). Under these conditions the maximum water velocity that would be expected of the water (ft/s) coming out of the basement faucet is most nearly:

(A) 23

(B) 34

(C) 45

(D) 50



02-06*. A fire hose nozzle has a diameter of 1.125 inches. According the local fire code, the nozzle must be capable of delivering at least 250 gpm when attached to a 3-in.-diameter hose. Under these conditions, the pressure (psig) that must be maintained just upstream of the nozzle is most nearly:

(A) 35

(B) 43

(C) 52

(D) 6,189

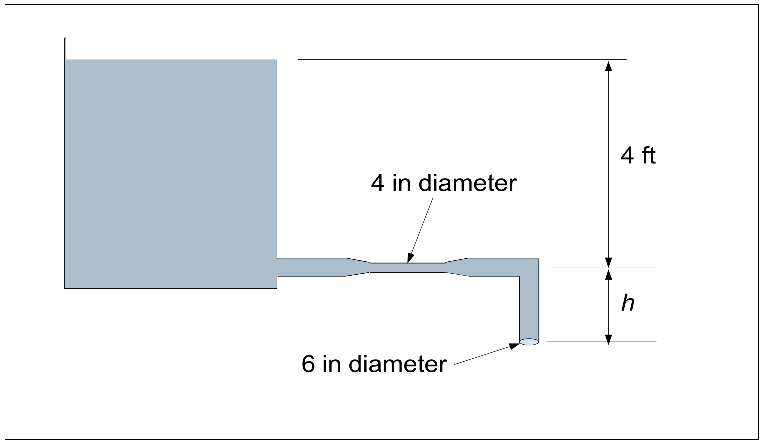
02-07*. Water flows steadily from a large tank, with negligible friction effects. The 4-in diameter section of the thin wall tubing will collapse if the pressure there drops to 10 psi below atmospheric pressure. Under these conditions, the largest allowable value of h (inches) is most nearly:

(A) 1.4

(B) 6

(C) 10

(D) 17



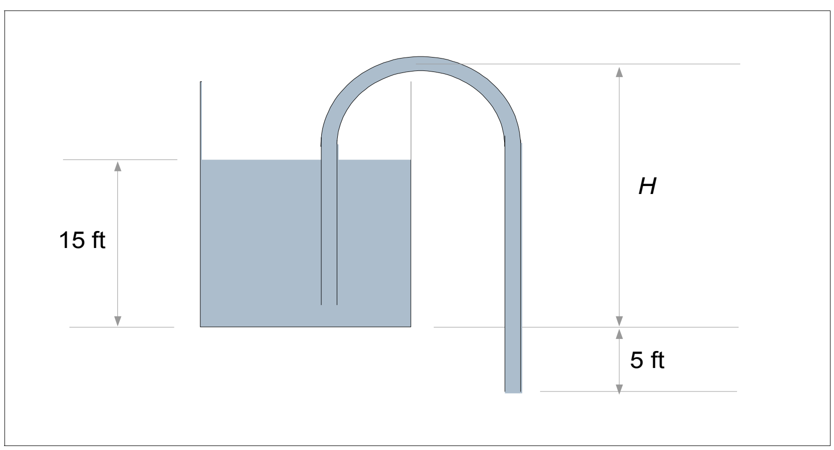
02-08*. Water at 60°F is being syphoned with a constant diameter hose from a large tank open to atmosphere, where the local atmospheric pressure is 14.7 psia. More information is provided in the picture. Neglecting all friction effects, the maximum height of the hill, H (ft) over which the water can be siphoned without cavitation (localized formation of water vapor bubbles in the hose due to evaporation) occurring is most nearly:

(A) 21

(B) 28

(C) 36

(D) 44



Section 03: Control Volume Analysis – Conservation of Mass & Energy

As engineers, our focus is typically on a device or a region in space into which a fluid enters and/or leaves. We identify this region as a control volume. For the purposes of the P.E. Exam, the most general form of the mass balance equation is:

$$\frac{dM}{dT}=\sum_{\mathrm{i}} \dot{m}_{\mathrm{in,\,i}}-\sum_{\mathrm{j}} \dot{m}_{\mathrm{out,\,j}} \tag{3-1}$$

That is, the rate at which the mass accumulates inside a control volume, dM/dT , is equal to total rate at which mass enters the control volume, $\Sigma \dot{m}_{\mathrm{in,\,i}}$ minus the total rate at which mass leaves the control volume, $\Sigma \dot{m}_{\mathrm{out,\,j}}$. We use the summation signs to account for multiple inlet and/or outlet ports.

When the total rates of mass into and out of the control volume are equal, there is no mass accumulation. This is the steady-state condition:

$$\sum_{\mathrm{i}} \dot{m}_{\mathrm{in,\,i}}=\sum_{\mathrm{j}} \dot{m}_{\mathrm{out,\,j}} \quad \text{(Steady State)} \tag{3-2}$$

If there is only one inlet and one outlet in the control volume, then:

$$\dot{m}_{\mathrm{in}}=\dot{m}_{\mathrm{out}} \quad \text{(Steady state, one inlet, one outlet)} \tag{3-3}$$

For incompressible fluids (liquids, or gases that do not experience significant density changes) the above equations can all be written in terms of volume and volumetric flow rates as well.

In incompressible Fluid Mechanics (and in the kinds of problems relevant to the PE exam) the most used form of the energy conservation equation is the so-called “extended” Bernoulli equation[7]. When applied to steady, incompressible, uniform flow with one inlet (labeled “1”) and one exit (labeled “2”) the energy equation becomes:

$$h_P+\frac{V_1^2}{2\mathrm{g}}+\frac{p_1}{\gamma}+z_1=h_T+\frac{V_2^2}{2\mathrm{g}}+\frac{p_2}{\gamma}+z_2+h_L \tag{3-4}$$

where h_P is known as the “**pump head**” (it has units of length) and is related to the amount of energy imparted on the fluid by the pump(s) in the control volume. Similarly, h_T is related to the amount of energy extracted from the fluid by the turbine(s). It can be shown that the power requirement by a pump with an efficiency η_P is:

7 An academic might “cringe” at this name. The Bernoulli equation is actually a momentum equation (F=ma) along a streamline. The energy equation is actually a statement of the conservation of energy principle. It just so happens that for the particular set of problems considered here, the two reduce to similar forms.

$$\dot{W}_P = \frac{\dot{m}\, g\, h_P}{\eta_P} \tag{3-5}$$

The numerator in equation (3-5) is the amount of energy the pump must impart on the fluid. Equation (3-5) shows the power required by the pump is slightly larger because some energy must be used to overcome friction and other irreversibilities within the pump.

Equation (3-5) uses the mass flow rate $\dot{m}$, which is not really that common in practice. It is far more common to use volumetric flow rate. Focusing now on the USCS, we define the "**Water Horse Power**" (WHP) as the power (in hp) that the pump adds to the water. This is also known as **hydraulic or theoretical horsepower**. The numerator in equation (3-5) is the water horsepower. In terms of volumetric flow rate, WHP can be calculated as:

$$\text{WHP} = \frac{h\langle \text{ft}\rangle Q\langle \text{gpm}\rangle SG}{3956} \tag{3-6}$$

Equation (3-6) only works if the flow rate is in gpm and the pump head is in ft. Another useful form of the WHP equation is:

$$\text{WHP} = \frac{\Delta p_{\text{pump}}\langle \text{psi}\rangle Q\langle \text{gpm}\rangle}{1714} \tag{3-7}$$

which can be used to calculate WHP if the pressure rise (in psi) across the pump is known as well as the flow rate (in gpm) through the pump. Equation (3-7) is obtained under the assumption that the change in velocity head across the pump is negligibly small when compared with the change in pressure head. The WHP is the power that the pump adds to the water. The power that the motor must deliver to the pump is the **brake horse power** (BHP), also known as the **shaft horse power**. The BHP is slightly larger than the WHP because some energy must be used to overcome friction and other irreversibilities. Therefore, for pumps:

$$\text{BHP}_P = \frac{\text{WHP}}{\eta_P} \tag{3-8}$$

Similarly, the shaft horsepower developed by a turbine of efficiency η_T is:

$$\text{BHP}_T = \eta_T\, \text{WHP} \tag{3-9}$$

where the turbine water horsepower is also calculated with equation (3-6).

The last term of equation (3-4) is the head loss, h_L . This is an umbrella term that covers the effects of viscosity-caused internal friction with its resulting temperature changes and heat transfer, as well as energy that must be expended to sustain secondary flow patterns that are created with geometry changes (elbows, tees, valves, etc). In pipe or conduit flow, the losses due to viscous effects are distributed over the entire length (these are known as **friction losses**), whereas the loss due to a geometry change is concentrated in the vicinity of the geometry change (these are known as **minor losses**). That is, $h_L = h_{minor} + h_{friction}$. The analytical calculation of these losses is quite complicated and instead we generally rely on empirical formulas to predict them. One common approach to estimate minor losses is to write the head loss in terms of a **head loss coefficient**, K so that:

$$h_{minor} = \left(\sum K\right) \frac{V^2}{2g} \tag{3-10}$$

In other words, the head loss is proportional to the velocity head, and the constant of proportionality is the sum of all the loss coefficients in the system. The head loss coefficient (as well as other aspects of the calculation of head losses such as friction losses) will be discussed in greater detail in further sections of this book.

"Time-saving" forms of common equations:

The formal definition of water horsepower was equation (3-5), but in problem-solving situations equations (3-6) and (3-7) are real "time-saver" alternatives because they are written in terms of commonly used quantities such as flow rate in gpm and pressure change in psi. This approach can be applied to many of the equations in this chapter to obtain these short-cut type equations. EXTREME CAUTION should be applied, however, and make absolutely sure you are using the units required to make the equations accurate:

$$h_V = 0.0155\,V^2 = 0.00259 \frac{(\text{gpm})^2}{D^4} \qquad [h_V] = \text{ft}\,;\; [V] = \frac{\text{ft}}{\text{s}};\; [D] = \text{in} \tag{3-11}$$

Equation (3-11) quickly provides h_V when the flow rate is known in gpm and the pipe ID is known in inches. This is useful because head losses are calculated as a loss coefficient times h_V .

To obtain flow velocity, given flow rate in gpm, use:

$$V = 0.4085 \frac{\text{gpm}}{D^2} \qquad [V] = \frac{\text{ft}}{\text{s}};\; [D] = \text{in} \tag{3-12}$$

Head and pressure are used interchangeably provided that they are expressed in their appropriate units. To convert from one to other use any of these four equations:

$$\text{Liquid Head in feet} = \frac{\text{psi} \times 2.31}{SG} \tag{3-13}$$

$$\text{Liquid Head in feet} = \frac{\text{psi} \times 144}{\gamma} \qquad [\gamma] = \frac{\text{lbf}}{\text{ft}^3} \tag{3-14}$$

$$\text{Pressure in psi} = \frac{\text{Head in ft} \times \gamma}{144} \qquad [\gamma] = \frac{\text{lbf}}{\text{ft}^3} \tag{3-15}$$

$$\text{Pressure in psi} = \frac{\text{Head in ft} \times SG}{2.31} \tag{3-16}$$

For water, it is typical to use $\gamma = 62.4\,\text{lbf}/\text{ft}^3$ – which corresponds to 60°F – in Equations (3-14) and (3-15). Also, Equations (3-13) and (3-16) show that a column of 60°F water 2.31 feet high will exert a pressure of 1 psi at its base.

PROBLEMS

03-01. Water flows at a rate of 400 gpm in a 4-in ID pipe. The pipe enters a tee from which a 2-in ID and a 3-in ID pipe come out. The flow velocity in the 3-in ID pipe is 8.7 feet per second. Calculate:

(a) the velocity head in the 4-in pipe upstream of the tee.

(b) the flow rate in gpm in the 3-in pipe downstream of the tee.

(c) the flow rate in gpm in the 2-in pipe downstream of the tee.

03-02. A certain valve in a 3-in pipe with 50 gpm of water causes a 1.0 psi pressure drop across the valve. Calculate:

(a) the head loss for the valve, in feet.

(b) the velocity head, in feet.

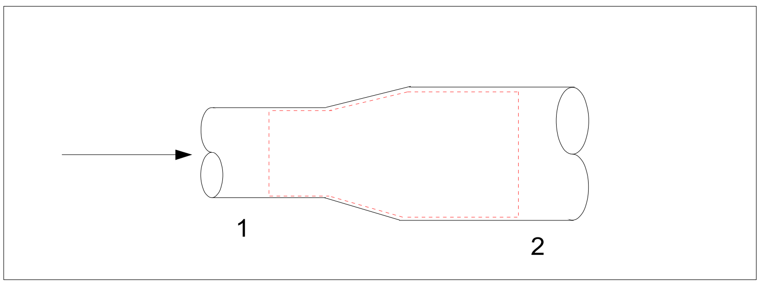
(c) the loss coefficient for the valve.

Now, assume the loss coefficient remains unchanged when the valve is placed in a 3-in pipe carrying 175 gpm of a heat transfer fluid with SG=0.75 – what pressure drop (in psi) will occur across the valve now?

03-03. A stream of water in a 6-in ID pipe feeds a shell-and-tube heat exchanger characterized by a loss coefficient of $K = 20$. The pressure drop across the heat exchanger is not to exceed 10 psi. What is the highest allowable flow rate (in gpm) to the heat exchanger?

03-04*. Water flows at a rate of 120 gpm in a seamless steel schedule 40, 2-in pipe (ID = 2.067 in). The static pressure is 10 psi just upstream of an expansion to 3-in pipe (ID = 3.068 in). If the loss coefficient associated with the expansion (based on the upstream velocity) is 0.37, the pressure (psi) just downstream of the expansion is most nearly:

(A) 10.4
(B) 15.8
(C) 23.8
(D) 71.7



03-05*. Water is transported from one atmospheric pressure reservoir with surface elevation of 440 feet to a lower atmospheric pressure reservoir with surface elevation of 80 feet through a 10-inch ID pipe. The flow rate is known to be 7,500 gallons per minute. Under these conditions, the total loss coefficient between the surfaces is most nearly:

(A) 15
(B) 25
(C) 30
(D) 45

03-06*. A pump with an efficiency of 85% raises the pressure of 80 gpm of water by 100 psi. The pipe at the pump suction has a diameter of 2 inches and the pipe at the pump discharge has a diameter of 4 inches. Under these conditions, the brake horsepower for the pump is most nearly:

(A) 3.0

(B) 3.5

(C) 4.7

(D) 5.5

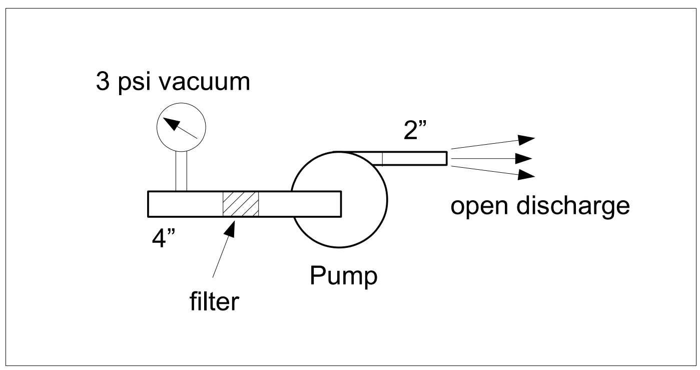
03-07*. The water pump shown has a brake horsepower of 30 hp and an efficiency of 90%. The flow rate is 790 gallons per minute and all piping on the suction line is 4-in ID pipe while the short discharge pipe is 2-in ID. The only significant head loss downstream of the pressure gage occurs across the filter. Under the conditions shown, the head loss (ft) associated with the filter is most nearly:

(A) 34

(B) 41

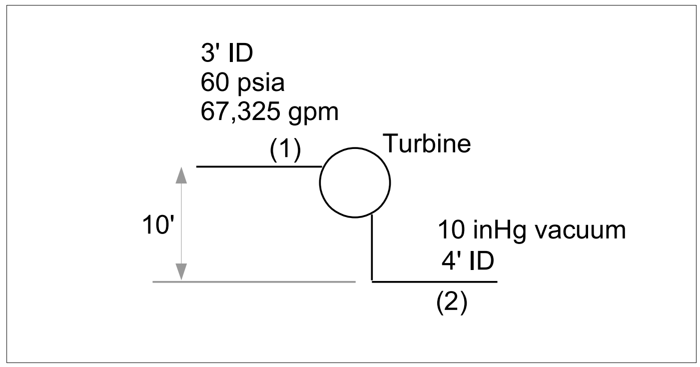
(C) 130

(D) 135



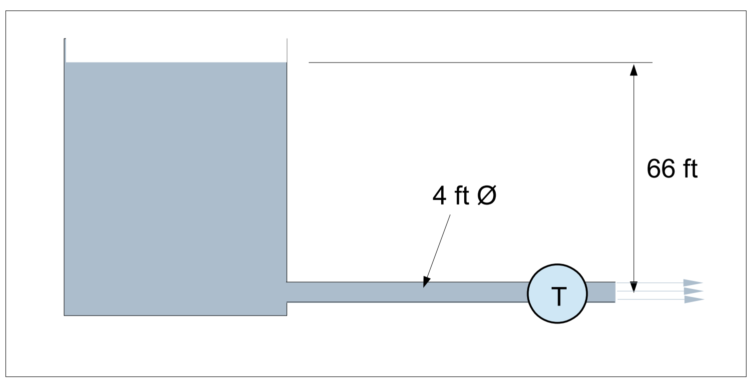
03-08*. Water at 60 psia flowing at a rate of 67,325 gallons per minute enters a hydraulic turbine through a 3-ft ID inlet pipe as indicated in the figure. The turbine discharge pipe has a 4-ft ID. The static pressure at section 2 (10 feet below the turbine inlet) is 10 in. Hg vacuum. The turbine has an efficiency of 85% and develops 2.25 MW at the shaft. The head loss (ft) between sections 1 and 2 is most nearly:

(A) 1.5
(B) 2.4
(C) 14
(D) 47



03-09*. A small river with a reliable flow rate of 15.25 million cubic feet per day feeds the reservoir shown in the figure. The turbine has a mechanical efficiency of 80% and the overall loss coefficient for the entire system is 4.5. Under these conditions, the power developed at the shaft, in kW is most nearly:

(A) 488
(B) 588
(C) 688
(D) 788



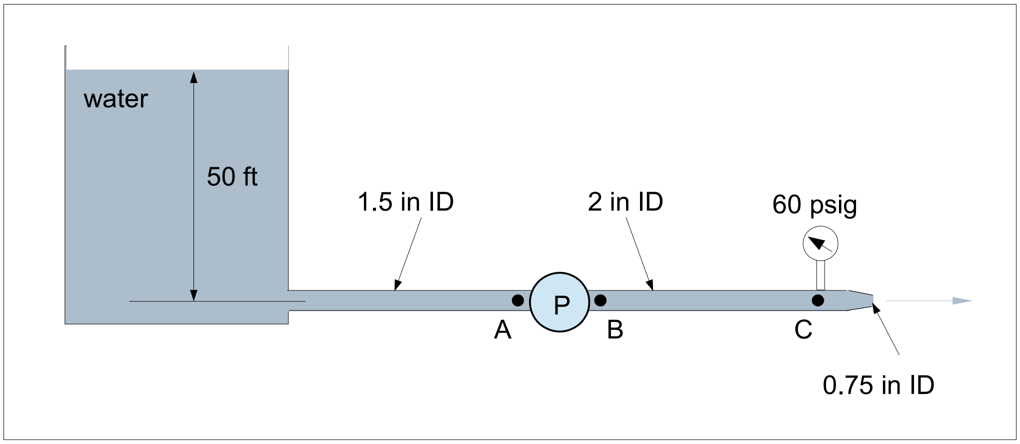
03-10*. The overall loss coefficient up to point A is 3.2; from B to C, the overall loss coefficient is 1.5, and the losses through the exit nozzle are negligible The figure shows additional information. Under these conditions, the power requirement (hp) for the 85% efficient pump is most nearly:

(A) 3.0

(B) 3.5

(C) 4.1

(D) 4.8



SLAY the P.E.

www.SlaythePE.com

