The mass conservation (or continuity) equation requires that for steady state:

$$\dot{m}_{\text{in}} = \dot{m}_{\text{out}} \tag{1}$$

The relationship between mass flow rate $\dot{m}$, and flow velocity V , is $\dot{m} = \rho \cdot A \cdot V$, therefore equation (1) can be re-written as:

$$\rho_{\text{in}} \cdot A_{\text{in}} \cdot V_{\text{in}} = \rho_{\text{out}} \cdot A_{\text{out}} \cdot V_{\text{out}} \tag{2}$$

Or in terms of the specific volume (the inverse of density):

$$\frac{A_{\text{in}} \cdot V_{\text{in}}}{v_{\text{in}}} = \frac{A_{\text{out}} \cdot V_{\text{out}}}{v_{\text{out}}} \tag{3}$$

Where $v_{\text{out}} = v_{f\ @\ 5.0\text{psia}} = 0.0164\,\text{ft}^3/\text{lbm}$. We now only need to find v_{in} . For a pressure of 5.0 psia, the saturation temperature is 162.2°F. With 50°F of superheat, the steam temperature is 212.2°F. From the steam tables, with 5.0 psia and 212.2°F we obtain $v_{\text{in}} \approx 79.6\,\text{ft}^3/\text{lbm}$.

Re-arrange equation (3) so that the area ratio (what we want to calculate) is on one side:

$$\frac{A_{\text{in}}}{A_{\text{out}}} = \frac{v_{\text{in}}}{v_{\text{out}}} \cdot \frac{V_{\text{out}}}{V_{\text{in}}} \tag{4}$$

Since the area is proportional to the square of the diameter, and using $V_{\text{in}}/V_{\text{out}} = 50$ from the problem statement we have:

$$\left(\frac{D_{\text{in}}}{D_{\text{out}}}\right)^2 = \frac{79.6}{0.0164} \cdot \frac{1}{50} = 97.1$$

Therefore, $D_{\text{in}}/D_{\text{out}} = 9.86$.